Math 22 - Calculus of Several Variables **Midterm 1 - Form A (white)**
M. Eastman - Spring 2008

1. a. Find the center and radius of the sphere given by the equation

$$x^2 + y^2 + z^2 + 12x - 8y - 18z - 17 = 0$$

b. Find the equation of the sphere where one of its diameters has endpoints $(5, -1, 2)$ and $(-1, 9, -12)$.

2. Given: $\boldsymbol{a} = < 9, -2, 6 >$ $\boldsymbol{b} = < 1, -4, 1 >$ $\boldsymbol{c} = < 2, 7, -1 >$

Find:

a. $\boldsymbol{b} \cdot (4\boldsymbol{a} - \boldsymbol{c})$ b. $|\boldsymbol{b} + 2\boldsymbol{c}|$ c. a <u>unit</u> vector in the opposite direction of $\boldsymbol{a}$.

d. $\boldsymbol{a} \times \boldsymbol{c}$ e. the angle formed by $\boldsymbol{b}$ and $\boldsymbol{c}$. f. $\text{comp}_{\boldsymbol{a}}(\boldsymbol{c})$ [the scalar projection of $\boldsymbol{c}$ onto $\boldsymbol{a}$]

3. Find the parametric equations of the line of intersection of the planes

$2x - y - 4z = 10$ and $x + 3y + 5z = 12$

4. Given the points $(2, -1, -1)$, $(1, 0, 1)$, and $(3, 2, 2)$:

a. Find the equation of the plane that passes through these points. Write your answer in the form $ax + by + cz + d = 0$.

b. Find the point at which the plane (from above) intersects the line whose symmetric equations are

$5 - x = \frac{y-7}{2} = \frac{z-9}{4}$

5. Identify the following quadric surfaces, and its axis of symmetry (x- y- or z-axis):

a. $25y^2 - z^2 = 4x^2 - 100$ b. $x + 9y^2 = 4z^2$

6. Find the domain of the vector function $\boldsymbol{r}(t) = < \frac{t-3}{t^2-16}, \sqrt{9-t}, ln(t+3) >$; write your answer using interval notation :

7. Given the curve defined by the vector function $\boldsymbol{r}(t) = < \sqrt{6t+3}, 4\,tan^{-1}(t), ln(2t-1) >$:
Find $\boldsymbol{r}'(t)$, $\boldsymbol{r}''(t)$, and $\boldsymbol{T}(1)$.

8. Given the vector function $\boldsymbol{r}(t) = < 4\,sin(2t), 3t, 4\,cos(2t) >$

a. Find the length of the curve from $t = 0$ to $t = \frac{\pi}{3}$.

b. find the curvature κ .

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1. a. center: $(-6,\ 4,\ 9)$ radius $= \sqrt{150} = 5\sqrt{6}$

   b. Center is at midpoint of given points, radius is one-half the distance between the points:

   $(x-2)^2 + (y-4)^2 + (z+5)^2 = 83$

2. a. 119 b. $\sqrt{126}$

   c. $< -\frac{9}{11}, \frac{2}{11}, -\frac{6}{11} >$ d. $< -40,\ 21,\ 67 >$

   e. $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ f. $-\frac{2}{11}$

3. a. $x = 6 + t$
   $y = 2 - 2t$ (there are many possible "initial" points)
   $z = 0 + t$

4. a. $3x - 5y + 4z - 7 = 0$ b. $(8,\ 1,\ -3)$

5. a. hyperboloid of one sheet, with axis of symmetry the y-axis.

   b. hyperbolic paraboloid, with axis of symmetry the x-axis.

6. $(-3,\ 4) \cup (4,\ 9]$

7. a. $\boldsymbol{r}'(t) = < \frac{3}{\sqrt{6t+3}},\ \frac{4}{1+t^2},\ \frac{2}{2t-1} >$

   $\boldsymbol{r}''(t) = < \frac{-9}{\left(\sqrt{6t+3}\right)^3},\ \frac{-8t}{(1+t^2)^2},\ \frac{-4}{(2t-1)^2} >$

   $\boldsymbol{T}(1) = < \frac{1}{3},\ \frac{2}{3},\ \frac{2}{3} >$

8. a. $\int_0^{\frac{\pi}{3}} \sqrt{(8\,cos\,2t)^2 + (3)^2 + (8\,sin\,2t)^2}\,dt = \int_0^{\frac{\pi}{3}} \sqrt{9 + 64(sin^2(2t) + cos^2(2t))}\,dt$

   $= \int_0^{\frac{\pi}{3}} \sqrt{73}\ dt = \left(\sqrt{73}\,t\right)\Big|_0^{\frac{\pi}{3}} = \frac{\pi\sqrt{73}}{3}$

   b. $\kappa = \frac{|\boldsymbol{r'}(t) \times \boldsymbol{r''}(t)|}{|\boldsymbol{r'}(t)|^3} = \frac{|\mathbf{T'}(t)|}{|\boldsymbol{r'}(t)|} = \frac{16}{73}$