Math 22 - Calculus of Several Variables **Midterm 2 - Form A**
M. Eastman - Spring 2008

1. Given $f(x,y) = 3 - \sqrt{y^2 - x}$:

   a. Sketch the the domain, and state the range of f.

   b. Sketch the level curves $f(x,y) = 0$ and $f(x,y) = 2$. Label each level curve clearly.

   c. Find the equation of the tangent plane to the surface when $x = 5$ and $y = -3$. Write your answer in the form $ax + by + cz + d = 0$.

2. Given $f(x,y) = sin(2y^2 - x) + ln(x^2 + 5y)$, find:

   a. $fy(x,y)$

   b. $f_{yy}(x,y)$

   c. $f_{xy}(x,y)$

   d. $\nabla f\,(\,2,\,-1)$

   e. the directional derivative of f at $(\,2,\,-1\,)$ in the direction of the point $(\,0,\,1\,)$.

3. Given $z = x^3 \cdot sin\, y$, where $x = v \cdot cos\, w$ and $y = \frac{w}{v}$: use the chain rule to find $\frac{\partial z}{\partial v}$ and $\frac{\partial z}{\partial w}$ when $v = 4$ and $w = \pi$.

4. Find the critical points for $f(x,y) = x^2 + xy^2 + 3y^2 + 2x + 24$. Find the value of f at each critical point, and classify the function value at each critical point as either a (local) maximum, (local) minimum, or a saddle point. The number of lines in the table do not necessarily indicate the number of points. Justify your answers.

5. Use Lagrange multipliers to find the extreme values of the function $f(x,y) = 2x^2 + 3y^2 + 4x - 5$ subject to the constraint $x^2 + y^2 = 16$, and the point(s) where these extremes occur.

6. Find $\int\int_{\mathbb{D}} (2y - x)\, dA$, where $\mathbb{D}$ is the region bounded by $y = x^2$ and $y = 2x$.

7. Switch the order of integration to evaluate: $\int_0^2 \int_{x^2}^4 x^3\, sin(y^3)\, dy\, dx$

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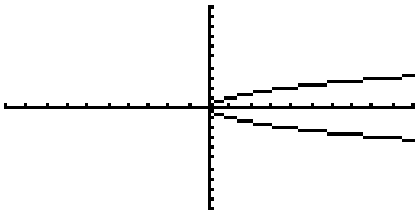
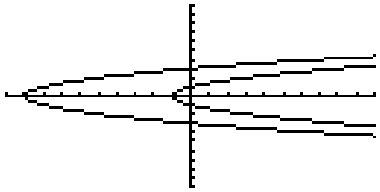
1. a. domain is region to the left of the parabola $x = y^2$



range $= (-\infty, 3]$

b. $f(x,y) = 0$ is parabola $x = y^2 - 9$
$f(x,y) = 2$ is parabola $x = y^2 - 1$



c. $x - 6y - 4z - 19 = 0$

2. a. $4y\,cos(2y^2 - x) + \frac{5}{x^2+5y}$ b. $4\,cos(2y^2 - x) - 16\,sin(2y^2 - x) - \frac{25}{(x^2+5y)^2}$

c. $4y\,cos(2y^2 - x) + \frac{10y-2x^2}{(x^2+5y)^2}$ d. $< -4, -9 >$ e. $-\frac{5\sqrt{2}}{2}$

3. $\frac{\partial z}{\partial x} = \left(24\sqrt{2}\right)(-1) + \left(-32\sqrt{2}\right)\left(-\frac{\pi}{16}\right) = 2\sqrt{2}(\pi - 12)$

$\frac{\partial z}{\partial y} = \left(24\sqrt{2}\right)(0) + \left(-32\sqrt{2}\right)\left(\frac{1}{4}\right) = -8\sqrt{2}$

4.

critical point	function value	type of point
$(-1, 0)$	23	minimum
$(-3, 2)$	27	saddle
$(-3, -2)$	27	saddle

5. $f_{max} = 47$ at $\left(2, \pm\sqrt{12}\right)$ $f_{min} = 11$ at $(-4, 0)$

6. $\int_0^2 \int_{x^2}^{2x} (2y - x)\,dy\,dx = \ldots = \frac{28}{5}$

7. $\int_0^2 \int_{x^2}^{4} x^3\,sin(y^3)\,dy\,dx = \int_0^4 \int_0^{\sqrt{y}} x^3\,sin(y^3)\,dx\,dy = \ldots = \frac{1-cos(64)}{12}$