8 Ratio and Proportion

8.1 Ratio

A ratio is a comparison between two quantities using division. The things being compared may be of the same type. For example, if Sara is playing basketball and makes 12 baskets out of the 19 shots she took, her ratio of baskets made to baskets attempted is \( \frac{12}{19} \) which is sometimes written \( 12 : 19 \) (This is read out loud as, “Twelve to nineteen”).

Sometimes ratios compare two different types of things. For example, if Jerry buys 8 pounds of apples for $16, then the ratio of dollars to pounds is \( \frac{16}{8} = \frac{2}{1} \) or 2. Notice that these ratios give us something more than just a way of writing the numbers in a fraction form. In the second case, \( \frac{16 \text{ dollars}}{8 \text{ pounds}} \), gives us the rate at which Jerry pays for apples, namely sixteen dollars for every 8 pounds or equivalently, $2 per pound.

**Exercise 8.1.1** Find the following ratios and simplify them whenever possible:

1. If you travel 400 miles in 8 hours, what is the ratio of miles to hours?
   
   What does this ratio tell you?

2. If you travel 352 miles on 16 gallons of gas, what is the ratio of miles to gallons?
   
   What does this ratio tell you?

3. If you work for 28 hours and make $392, what is the ratio of dollars to hours?
   
   What does this ratio tell you?

4. If you make $50,000 in 12 months, what is the ratio of dollars to months?
   
   What does this ratio tell you?

5. A 2 foot stick measures 24 inches. What is the ratio of inches to feet?
   
   What does this ratio tell you?

6. A 20 inch stick measures 50.8 centimeters. What is the ratio of centimeters to inches?
   
   What does this ratio tell you?
Exercise 8.1.2 Use information from your class to find the ratios. Write the ratios as fractions in simplest form.

1. The ratio of females to males in your math class.
2. The ratio of males to females in your math class.
3. The ratio of females to all people in your math class.
4. The ratio of males to all people in your math class.
5. The ratio of smokers to nonsmokers in your math class.
6. The ratio of nonsmokers to smokers in your math class.
7. The ratio of smokers to all people in your math class.

8.2 Equivalent Fractions

We will often find it useful to be able to generate equivalent fractions with a particular denominator. For example, if we want to write $\frac{5}{8}$ as a fraction out of 56 rather than 8, we will need a procedure. Let’s see if we can describe how to do this.

Exercise 8.2.1

1. Find the missing numbers in the fractions below.

   (a) $\frac{3}{5} = \frac{20}{\boxed{}}
   \quad$  (b) $\frac{4}{9} = \frac{54}{\boxed{}}
   \quad$  (c) $\frac{1}{12} = \frac{60}{\boxed{}}
   \quad$  (d) $\frac{7}{13} = \frac{91}{\boxed{}}$

2. Describe your procedure for finding the missing numbers in the fractions above.

3. Use your procedure in (2) to help you find the missing number in the fraction: $\frac{4}{5} = \frac{\boxed{}}{42}$
8.3 Proportion

When two ratios are equal it is called a proportion.

**Example:** Suppose that the ratio of boys to girls at a local high school, \(\frac{882}{1029}\), is equivalent (after simplifying) to the ratio of boys to girls in an algebra class \(\frac{6}{7}\). Then we say that the ratio of boys to girls at the school is in proportion to the number of boys and girls in the class \(\frac{882}{1029} = \frac{6}{7}\). Similarly suppose the ratio of boys to girls in Cleveland is in proportion to the number of boys and girls in that class. If there are 126,000 girls in Cleveland, how many boys are there?

We would write \(\frac{\text{# boys in class}}{\text{# girls in class}} = \frac{\text{# boys in Cleveland}}{\text{# girls in Cleveland}}\)

\[\frac{6}{7} = \frac{?}{126000}\]

Since \(\frac{126000}{7} = 18000\), you multiply the numerator by 18,000:

\[\frac{6 \times 18000}{7 \times 18000} = \frac{108000}{126000}\]

So there are 108,000 boys in Cleveland.

**Exercise 8.3.1** Use a proportion to solve each problem.

1. A car travels 224 km on 4 gallons of gas. How far can it be expected to travel on a tank of 12 gallons?

2. If you make $600 (after taxes) working for 70 hours, how much will you make working for 500 hours?

3. A 425 pound motorcycle weighs 68 pounds on the moon. How much will a 120 pound woman weigh on the moon?

   How much would you weigh on the moon?

4. 1 inch is equivalent in length to 2.54 centimeters. How tall (in inches) is someone who is 180cm?

5. A recipe for six dozen cookies calls for \(2\frac{1}{2}\) cups of flour. How many cups of flour are needed for 10 dozen cookies?

6. The floor plan of a house is drawn to the scale of \(\frac{1}{4}'' = 1'\). The master bedroom measures \(3\frac{1}{2}''\) by \(5''\) on the blueprints. What is the actual size of the room?

7. On average, an adult flea is 3mm long but it can long jump 330mm (about 13 inches) and high jump 204mm (about 8 inches). If you could jump in proportion with a flea, how far could you long jump? High jump?

8. If a company’s stock falls by 3% or $1.20, how much was the stock worth originally?
9. a) If one gallon of paint covers 400 square feet, how many gallons of paint do you need to cover 5000 square feet?

    b) If one gallon of paint covers 400 square feet, how many square feet will 27 gallons cover?

10. If your car gets 25 miles to the gallon and gas costs $3.40 per gallon, how much will a 420 mile trip cost?

### 8.4 Percent

Percent is a special case of proportion. The word comes from the Latin for *by the hundred* and it is used to represent fractions as their equivalent value out of 100. For example, since $\frac{1}{5}$ is equivalent to $\frac{20}{100}$, we say that $\frac{1}{5}$ is the same as 20 percent. To remind us that a percent is a fraction of 100, the percent symbol is composed of the division sign and the two zeros we find in 100: %.

#### Example 1:

What percent of the rectangle below is shaded?

**Solution 1:** Since there are 20 boxes and 9 of them are shaded, we can say $\frac{9}{20}$ of the rectangle is shaded. To write this as a percentage, we need to find an equivalent fraction out of 100:

\[
\frac{9}{20} = \frac{9 \times 100}{20 \times 100} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100}
\]

So 45% of the rectangle is shaded.

**Solution 2:** One great thing about wanting to know how many hundredths that you have, is that the hundredths place is one of our place values! If we can turn $\frac{9}{20}$ into a decimal by dividing, then we can see the answer:

\[
\frac{9}{20} = 9 \div 20 = 0.45 \text{ which is } 45 \text{ hundredths, or } 45\%.
\]
Example 2:
What percent of the rectangle below is shaded?

Solution: Since there are 40 boxes and 15 of them are shaded, we can say \( \frac{15}{40} \) of the rectangle is shaded. To write this as a percentage, we need to find an equivalent fraction out of 100:

\[
\frac{15}{40} = \frac{15 \times 100}{40 \times 100} = \frac{15 \times 2.5}{40 \times 2.5} = \frac{37.5}{100}
\]

So 37.5% of the rectangle is shaded.

Exercise 8.4.1
What percent of each rectangle is shaded?

(1) (2) (3)
Example:

Shade 30% of the rectangle shown below.

In order to shade 30% of the rectangle, we need to know what fraction of the total squares to shade. Since there are 20 squares in all we need to write 30% as a fraction out of 20:

\[
\frac{30}{100} = \frac{6}{20}
\]

So we shade six squares:

Exercise 8.4.2

1. Shade the indicated percentage of each rectangle.
   (a) 15%  
   (b) 40%  
   (c) 67%

2. Show three different ways of shading 50% of a rectangle
   (a)  
   (b)  
   (c)
8.5 Proportion and Percent in Problem Solving

Since a percent is really just a fraction, one way of taking a percentage of something is the same as taking a fraction of it. For example, 60% of 20 is the same as writing $\frac{60}{100} \times 20 = \frac{1200}{100} = 12$.

In addition, we can solve problems involving percentages by setting up and solving proportions.

**Example 1:**

There are 30 students in class, and you are told that 40% of them are male. How many males are there?

**Solution:**

We can set up a proportion by equating the males to students ratio. Knowing that 40% of the class is male, means that the males to students ratio is 40 to 100. We want to know how many males there are if there are 30 students. The proportion looks like the following:

$$\frac{40 \text{ males}}{100 \text{ students}} = \frac{\text{unknown number of males}}{30 \text{ students}}$$

To solve the proportion, we use the golden rule. To find out what to multiply times 100 to get 30, we divide $30 \div 100 = 0.3$ and multiply the numerator and denominator of $\frac{40}{100}$ by 0.3 to obtain the following:

$$\frac{40 \text{ males} \times 0.3}{100 \text{ students} \times 0.3} = \frac{12 \text{ males}}{30 \text{ students}}.$$ There are 12 males in the class.

**Example 2:**

During one year, the Math Club had 14 female members. That was 70% of the students in the club. What was the total number of students in the Math Club that year?

**Solution:**

Again, we can set up a proportion. Knowing that 70% of the club is female, means that the females to students ratio is 70 to 100. This time, the unknown is the number of students when there are 14 females. The proportion looks like the following:

$$\frac{70 \text{ females}}{100 \text{ students}} = \frac{14 \text{ females}}{\text{unknown number of students}}$$

To solve the proportion, we use the golden rule. To find out what to multiply times 70 to get 14, we divide $14 \div 70 = 0.2$ and multiply the numerator and denominator of $\frac{70}{100}$ by 0.2 to obtain the following:

$$\frac{70 \text{ females} \times 0.2}{100 \text{ students} \times 0.2} = \frac{14 \text{ females}}{20 \text{ students}}.$$ There are 20 students in the class.
Exercise 8.5.1 Solve the following ratio, proportion, and percent problems.

1. Find the following percentages.
   (a) 25% of 60   (b) 30% of 40   (c) 12% of 40

2. Some special percentages. Find the following percentages.
   (a) 10% of 70   (b) 10% of 120   (c) 10% of 35   (d) 10% of 19

3. What patterns do you notice about your answers in the previous exercise?

4. 21 out of 30 students commute more than 10 miles each day. What percent commute more
   than 10 miles? (Hint: The unknown is how many out of 100.)

5. Tax around California is about 8%. Estimate tax on each item by using 10% as your
   estimate and then add this to estimate the total cost.
   (a) $50 shirt   (b) $75 jacket   (c) $234 camera   (d) $1.75 coffee

6. Determine what amount the following numbers are 10% of. (e.g. $35 is 10% of $350).
   (a) $8 (Careful! the answer is NOT $0.80!)   (b) $29   (c) $43.50
   (d) $0.60

7. Some more special percentages. Find the following percentages in your head.
   (a) 20% of 70   (b) 20% of 120   (c) 20% of 35   (d) 20% of 19

8. Some more special percentages. Find the following percentages in your head.
   (a) 1% of 70   (b) 1% of 120   (c) 1% of 35   (d) 1% of 19

9. A sweater that normally costs $40 is marked down by 20%. How much will it cost now
   (before tax)?

10. Jimmy had $120 to spend on a VCR. What price of VCR can he afford if the tax, shipping,
    and handling comes to 18% of the price of the VCR?

11. Albert is making Chinese food for his friends who are coming over for dinner Saturday
    night. The recipe is asking for $5\frac{1}{2}$ cups of mushrooms, $3\frac{3}{4}$ cups of bell peppers, and $2\frac{1}{4}$
    cups of chicken breast. This recipe will feed 8 people. Albert only wants to feed 5 people.
    How many cups of each ingredient does he need?

12. If 45% of the workers in Jack’s company “brown bag” their lunch, how many people are
    in the company if 57 people “brown bag” their lunch?
Exercise 8.5.2 Continue to solve more ratio, proportion, and percent problems.

1. A new car is offered with a 12% rebate. If the car normally costs $18,000, how much will it cost now (before tax)?

2. Of the 1600 students who voted, 45% of them voted yesterday. How many students voted yesterday?

3. 9 out of 16 people voted for the “Clean Air” bill in Clark’s town. If 15,000 people voted for the “Clean Air” bill, how many people are there in Clark’s town?

4. If a computer costs $1000 but is marked down 30%, how much will it cost with 8.25% tax?

5. Suppose you have $35 in your wallet and you go out to eat. If you plan to pay 20% of the bill in tip and taxes, how much should your meal add up to without tip and taxes in order for you to cover the bill?

6. There are 65 students in Biology 101. If 43 students went on the field trip yesterday, what percent of the students didn’t go on the field trip yesterday?

7. Anderson and Jake went fishing. Anderson caught 22 fish. That amount was 89% of the number of fish that Jake caught. How many fish did Jake catch?

8. There are 26 students in your chemistry class. Eight students were absent on Friday. What percent of the students were absent? What percent of the students were present?

9. Sammy bought a jacket that was on sale for 85% of the original price. If Sammy paid $240, what was the original price of the jacket?

10. Mehdi bought 15 feet of pipe from the hardware store. How many yards of pipe did he buy if each yard equals three feet?

11. Nikkie bought a pair of shoes with a purchase price of $43. The sales tax in her county is 8.25%. What was her total cost?

12. If you put 9 red marbles in a bag with six green marbles,

(a) what percent of the marbles are red?

(b) what percent of the marbles are green?

13. If Michelle makes 63% of her shots in basketball, how many shots did she make if she attempted 40 shots?
8.6 Unit Conversion

Unit conversions can be computed using proportions since in essence a unit conversion factor is a ratio. For example, we know that there are 12 inches in a foot. This means that the inches to feet ratio is 12 to 1!

Example 1:

There are 12 inches in each foot. What is the measure in feet of a wall that measures 162 inches?

Solution:

We can set up a proportion by equating the inches to feet ratios. The proportion looks like the following:

\[
\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{162 \text{ inches}}{\text{unknown number of feet}}
\]

To solve the proportion, we use the golden rule. To find out what to multiply times 12 to get 162, we divide \(162 \div 12 = 13.5\) and multiply the numerator and denominator of \(\frac{12}{1}\) by 13.5 to obtain the following:

\[
\frac{12 \text{ inches} \times 13.5}{1 \text{ foot} \times 13.5} = \frac{162 \text{ inches}}{13.5 \text{ feet}}.
\]

The wall measures 13.5 feet.

Exercise 8.6.1 Compute the following unit conversions using proportions.

1. After working 3500 hours without any time off, Jerry decided to take a vacation.
   (a) If, on average, Jerry worked 8 hours a day, how many days has he worked?
   (b) If Jerry worked 5 days each week, how many weeks has he worked?
   (c) How many minutes has he worked?
   (d) How many seconds has he worked?

2. The radius of the earth is 6307 kilometers.
   (a) If 1 mile equals 1.609 kilometers, what is the radius of the earth in miles?
   (b) If 1 kilometer equals 0.62 miles, what is the radius of the earth in miles?
   (c) If 1 kilometer equals 1000 meters, what is the radius of the earth in meters?
   (d) If 1 mile equals 5280 feet, what is the radius of the earth in feet?
   (e) If 1 meter equals 100 centimeters, what is the radius of the earth in centimeters?
   (f) If 1 foot equals 12 inches, what is the radius of the earth in inches?
3. José is carpeting a room that measures 12 feet 9 inches by 15 feet.

   (a) How many square feet of carpet does he need?
   (b) How many square inches of carpet does he need?
   (c) How many square yards of carpet does he need?
   (d) If 1 inch equals 2.54 centimeters, how many square centimeters of carpet does he need?
   (e) How many square meters of carpet does he need?

4. Alice is building a patio in her backyard. There are a variety of tiles that she can choose from. If the patio measure 8 yards by 12 yards. How many tiles does she need if the size of the tiles are:

   (a) 1 square yard?
   (b) 1 square foot?
   (c) 1 square inch?
   (d) 1 square meter?
   (e) 1 square centimeter?

5. You are traveling around the world and want to spend $1200 in every country that you visit. Use the internet or a newspaper to find the current exchange rates for different currencies, then find out how much money you would have to spend in the currencies of the following countries:

   (a) Mexico.
   (b) The Philippines.
   (c) United Kingdom.
   (d) France.
   (e) China.
   (f) Japan.
In the following activity, you will learn how to calculate your course grade using a Simple Average, and then continue to learn how to compute a Weighted Average.

An *Average* is a number which summarizes several other numbers. It gives information about a group of numbers using a single value.

To calculate the Simple Average of a list of numbers,

- Find the sum of the list of numbers.
- Count how many numbers are in the list.
- Divide the sum by the count.

**Exercise 8.6.2**

1. Suppose a student got a 70 on the first exam, an 80 on the second exam, and a 90 on the third exam.
   - (a) What is the sum of all the scores?
   - (b) What is the count of all the scores?
   - (c) What is the Simple Average of all the scores?

2. Now suppose a student got a 60 on the first exam, an 80 on the second exam, and a 100 on the third exam.
   - (a) What is the sum of all the scores?
   - (b) What is the count of all the scores?
   - (c) What is the Simple Average of all the scores?

3. Finally, suppose a student got an 80 on the first exam, an 80 on the second exam, and an 80 on the third exam.
   - (a) What is the sum of all the scores?
   - (b) What is the count of all the scores?
   - (c) What is the simple average of all the scores?

4. Explain how a student could get different scores, but have the same Simple Average.

5. Find the simple average of the following exam scores: 93, 85, and 76.

6. Find the simple average of the following quiz scores: 15, 10, and 19.

7. Suppose each quiz was out of a possible 20 points. Find the percent correct for each of the quiz scores: 15, 10, and 19.

8. Find the simple average of the percents in the previous problem.
9. Find the simple average of the following project scores: 45, and 48.

10. Suppose a student has already scored 93, 85, and 76 on the first three exams. What is the least number of points they need on the fourth exam to have a “B” (at least 80) average?

11. Explain in complete sentences how you were able to calculate the last problem.

12. Before taking the fourth exam, you are curious to see if an “A” average is possible. How would you investigate the possibility, and what is your conclusion?

A Weighted Average counts certain scores more than others. Many instructors use a weighted average to calculate your grade because they wish to weigh your exam performance more heavily than, say, your group quizzes.

Example 1: Suppose your instructor weighs exams twice as much as group quizzes. Then, suppose you have taken 1 exam and scored 50%, and one group quiz and scored 90%. Find the simple average of the two percents, then find the weighted average.

Solution: The simple average is (simply!) \( \frac{50 + 90}{2} = 70 \). The simple average is 70%.

For the weighted average, we multiply each score by its weight before adding, then divide by the total of the weights. Since exams should be weighted twice as much as group quizzes, we will use 2 for the exam weight, and 1 for the group quiz weight. The weighted average is then:

\[
\frac{2 \cdot 50 + 1 \cdot 90}{2 + 1} = \frac{190}{3} \approx 63.33.
\]

The weighted average is approximately 63.33%.

Example 2: Suppose your instructor weighs exams as 60%, quizzes as 20%, and homework and projects as 10% each. Suppose that your simple averages in each category are as follows:

- Exams: 67%
- Quizzes: 72%
- Homework: 80%
- Projects: 70%

Is this student passing with a weighted average of over 70%?

Solution: The weighted average is calculated as follows:

\[
\frac{60 \cdot 67 + 20 \cdot 72 + 10 \cdot 80 + 10 \cdot 70}{60 + 20 + 10 + 10} = \frac{4020 + 1440 + 800 + 700}{100} = \frac{6960}{100} = 69.6
\]

The student is barely not passing. They have a weighted average of 69.6%.
Exercise 8.6.3 Answer the following questions based on the given information:

- The instructor uses the following weights: 50% for exams, 25% for quizzes, 15% for projects, and 10% for group activities.
- The student has the following exam percentages: 83, 91, 75
- The student has the following quiz percentages: 78, 65, 92, 81
- The student has the following project percentages: 95, 89
- The student has the following group activity percentages: 79, 84, 68, 87

1. What is the simple average of the exams?
2. What is the simple average of the quizzes?
3. What is the simple average of the projects?
4. What is the simple average of the group activities?
5. What is their overall weighted average?
6. If the scores in the other categories stayed the same, what exam average would this student need to have a weighted average of 90%?
7. If there are two more exams but no more assignments in the other categories left, is it possible for this student to have a weighted average of 90%?

At Skyline College, your overall GPA is a weighted average where the scores are numbers corresponding to the grade in the course, and the weights are the number of units in the course. The numbers for the grades are: A = 4, B = 3, C = 2, D = 1, and F = 0.

Exercise 8.6.4 Answer the following GPA questions.

1. During their first semester, a student takes American History (3 units) and gets a B, Philosophy (3 units) and gets a C, and Fundamentals of Math (5 units) and gets an A. What is their overall GPA for this semester?
2. The next semester, they take English (3 units) and get a B, PE (0.5 units) and get an A, the first half of beginning algebra (3 units) and get a B, and an MS Word class (2 units) and get a C. What is their GPA for this semester?
3. What is their overall combined GPA for the two semesters?
4. They hear that a scholarship is available, but the GPA requirement is a 3.5. How many units of straight A work would they need in order to boost their GPA to a 3.5?
9 Order of Operations

Discussion Questions

1. At Santa Cruz Beach Boardwalk, I bought an All-Day Unlimited Rides Wristband for $26.95, and 3 cotton candies for $1.25 each.

   (a) What two different operations (addition, subtraction, multiplication, division) could be used to calculate the total cost for my day at The Boardwalk?

   (b) Write in words how to calculate the total cost, including what order you use the numbers given in the problem and operations from part (a).

2. Each month, the household spends $450 for food and $1950 for rent.

   (a) What two different operations (addition, subtraction, multiplication, division) could be used to calculate the total yearly budget for food and rent?

   (b) Write in words how to calculate the total yearly budget for food and rent, including what order you use the numbers given in the problem and operations from part (a).

9.1 Order of Operations Agreement

Whenever there is more than one operation used to solve a problem, it is important to know which order to calculate. If you calculate in the wrong order, you will get the wrong answer! Since the order makes a difference, mathematicians came to an agreement (the Order of Operations Agreement) about how to indicate what order the operations must be performed. For multiplication and addition the agreement is:

- Multiplication must be performed before addition, regardless of the order it is written.
- If the addition should be performed first, use grouping symbols (parenthesis).

**Example 1:** Evaluate \(3 + 5 \times 2\).

**Solution:** The multiplication is performed first, so \(3 + 5 \times 2 = 3 + 10 = 13\).

**Example 2:** How can we make 3 plus 5 times 2 equal to 16?

**Solution:** To indicate that the addition should be performed first, use grouping symbols: \((3 + 5) \times 2 = 8 \times 2 = 16\). (Note: When parenthesis are used, and a number is multiplied by the result in the parenthesis, the \(\times\) symbol is optional. The previous expression could have been written: \((3 + 5)2\) or more commonly, \(2(3 + 5)\).)