5.5 Subtracting Fractions with Like Denominators

The idea of subtracting fractions is the same as with addition. For example, if you have three quarters and take away two quarters, you have one quarter left. Back in the early days that would look like:

\[
\frac{3}{4} - \frac{2}{4} = \frac{1}{4}
\]

With rectangles representing each whole, a representation of this problem might look like:

\[
\frac{3}{4} \quad \text{rectangle} \quad \frac{2}{4} \quad \text{rectangle} \quad \frac{1}{4} \quad \text{rectangle}
\]

Or, on a ruler if you start at \(\frac{3}{4}\) and jump back \(\frac{2}{4}\), you will land at \(\frac{1}{4}\).

Notice that just like with addition, the denominators must be the same in order for the problem to be possible.

Just as important, when we represent the subtraction problem with a picture, we must remember to make the rectangles representing each whole the same size and shape, and then make sure the fractional pieces within each whole are the same size and shape as well.
Exercise 5.5.1 For each of the following differences:

(a) Draw a picture using shaded rectangles representing the difference.
(b) Represent the difference on the given ruler.
(c) Find the result of the difference.

1. $\frac{3}{4} - \frac{1}{4}$
   
   (a) Drawing with rectangles:

   (b) Representation on a ruler:

   ![Ruler Representation]

   (c) The result:

2. $\frac{5}{8} - \frac{3}{8}$

   (a) Drawing with rectangles:

   (b) Representation on a ruler:

   ![Ruler Representation]

   (c) The result:
3. \( \frac{3}{2} - \frac{1}{2} \)

(a) Drawing with rectangles:

(b) Representation on a ruler:

(c) The result:

4. \( \frac{5}{4} - \frac{3}{4} \)

(a) Drawing with rectangles:

(b) Representation on a ruler:

(c) The result:
5. \( \frac{7}{2} - \frac{1}{2} \)

(a) Drawing with rectangles:

(b) Representation on a ruler:

(c) The result:

6. \( \frac{7}{4} - \frac{1}{2} \)

(a) Drawing with rectangles:

(b) Representation on a ruler:

(c) The result:
7. \( \frac{7}{8} - \frac{1}{2} \)

(a) Drawing with rectangles:

(b) Representation on a ruler:

(c) The result:

Some of the answers in the last exercise could be written as an equivalent fraction with a smaller denominator. For the problems in which this is true, check "yes", and write the original difference as well as the simpler, equivalent fraction. If there is no simpler, equivalent fraction, check "no". The second one is done for you.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Yes</th>
<th>No</th>
<th>Original Fraction</th>
<th>Equivalent Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>√</td>
<td></td>
<td>( \frac{2}{8} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<td>5</td>
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<td>6</td>
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<td>7</td>
<td></td>
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</tbody>
</table>
5.6 Comparing Fractions

How can we decide which fraction is bigger?

- If the fractions have the same denominator, then the size of the pieces is the same, and we are just comparing the number of pieces.

<table>
<thead>
<tr>
<th>Example 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td>We can compare $\frac{3}{4}$ and $\frac{2}{4}$ by seeing that $\frac{3}{4}$ has one more fourth than $\frac{2}{4}$ does:</td>
</tr>
<tr>
<td>$\frac{3}{4} &gt; \frac{2}{4}$</td>
</tr>
</tbody>
</table>

- If the fractions do not have the same denominators, we need to find equivalent fractions for each that do have the same denominator:

<table>
<thead>
<tr>
<th>Example 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>How can we compare $\frac{5}{8}$ and $\frac{1}{2}$? By seeing that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$, we can see that $\frac{5}{8}$ is bigger:</td>
</tr>
<tr>
<td>$\frac{5}{8} &gt; \frac{4}{8}$</td>
</tr>
</tbody>
</table>
Exercise 5.6.1 For the following pairs of fractions,

(a) Draw a picture for each to figure out which is bigger. (b) Place a $>$, $=$, or $<$ symbol between the fractions to indicate their relative size.

1. $\frac{3}{4}$ and $\frac{1}{4}$
2. $\frac{5}{8}$ and $\frac{7}{8}$
3. $1\frac{1}{3}$ and $\frac{2}{3}$
4. $\frac{3}{4}$ and $\frac{3}{2}$
5. $\frac{1}{2}$ and $\frac{3}{8}$
6. $\frac{1}{4}$ and $\frac{3}{8}$
7. $\frac{3}{4}$ and $\frac{5}{8}$
8. $1\frac{5}{8}$ and $1\frac{3}{4}$

Recall: Adding Fractions on a Ruler

Another way to visualize which fraction is bigger is to think of them as lengths. Recall that when we were using the rulers to visualize the addition of fractions, we identified where the first fraction sat on the ruler, and then added the second fraction by jumping ahead a length equivalent to the other fraction. For example, to illustrate $\frac{1}{2} + \frac{3}{4}$, we drew a picture similar to the following:

\[
\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}
\]

Comparing Fractions on a Ruler

To compare, just place each fraction on the ruler, and see which one is a longer length!

\[
\frac{1}{2} \quad \frac{3}{4}
\]
Exercise 5.6.2

For the following pairs of fractions,

- Indicate where each fraction lies on the ruler to see which is bigger.
- Place a $>$, $=$, or $<$ symbol between the fractions to indicate their relative size.

1. $\frac{11}{8}$ and $\frac{11}{4}$

2. $\frac{3}{3}$ and $\frac{3}{8}$

3. $\frac{5}{16}$ and $\frac{3}{8}$

4. $\frac{3}{4}$ and $\frac{2}{5}$

5. $\frac{1}{2}$ and $\frac{7}{16}$
5.7 Subtraction in Real Life

One of the first places you will encounter subtraction after this class is...in your next math class! The most common words that are used to imply subtraction in math problems are “difference between” and “subtracted from”. Because the order that numbers are subtracted changes the difference (unlike addition in which order doesn’t matter), it is important to know what order is implied.

Example 1: What is the difference between $3\frac{1}{8}$ and $2\frac{1}{2}$?

Solution:

We can find the difference by subtracting $3\frac{1}{8} - 2\frac{1}{2}$:

From previous problems and pictures, we have seen that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. We therefore break up the $\frac{1}{2}$ in $2\frac{1}{2}$:

So, our difference is equivalent to $3\frac{1}{8} - 2\frac{4}{8}$. Because 4 eighths is more than 1 eighth, we have to borrow 8 eighths from the third whole. Now $3\frac{1}{8}$ is equivalent to $2\frac{9}{8}$:
Without the pictures, this process looks like:

\[
\begin{array}{cc}
2 & \frac{1}{8} \\
- & \frac{4}{8} \\
\hline
\end{array}
\]

\[
\begin{array}{cc}
2 & \frac{9}{8} \\
- & \frac{4}{8} \\
\hline
\frac{5}{8} \\
\end{array}
\]

So, the difference between \(3\frac{1}{8}\) and \(2\frac{1}{2}\) is \(\frac{5}{8}\).

**Example 2:** What is 179.95 subtracted from 438.50?

**Solution:**

When the words “subtracted from” are used, the order that the numbers appear is opposite the order that they are subtracted. 179.95 subtracted from 438.50 is equivalent to the difference between 438.50 and 179.95 which is 258.55. (Verify this! Math books often leave out details in an example if it’s something that they think you have already done. For practice and to make sure, you should always go through on your own any details that are left out.)

So, 179.95 subtracted from 438.50 is 258.55.
In a more real life setting, there are three common situations where subtraction is used. They are finding a difference in two quantities, finding how much more you need, and finding the change in a quantity.

**Difference:** If you want to know how far apart two quantities are, literally find the difference in height, weight, price, bank balance, or anything that has size, subtract the smaller quantity from the larger quantity.

**Example 1:** At their 5 year check-up, the doctor weighed each twin. The older twin weighed 42.3 lbs, while the younger twin weighed 33.5 lbs. How much heavier is the older twin than the younger twin?

**Solution:** It is common in math problems, that the idea is the same as something you’ve studied, but the words used to describe the idea are different. You must practice the process of understanding the given words, by reading the passage many times and thinking clearly about what it means, then thinking of what familiar math idea that it is equivalent to. Only through practice, and asking questions if a certain problem doesn’t make sense, will you get good at this. In this case, although the word “difference” is not used in the problem, the given question is equivalent to the question, “What is the difference in their weights?” The difference in their weights is:

\[ 42.3 - 33.5 = 8.8 \]

So, the older twin is 8.8 lbs heavier than the younger twin.

**How Much More:** If you have a certain amount of something and want to know how much more you need to get to a total, subtract how much you have from the total that you want. The difference is how much more you need.

**Example 2:** A math teacher is saving to buy a 2002 Prius 4-door which costs $15,998.95. He has already saved $6,362.17. How much more does he need to save?

**Solution:** This time the wording in the problem is identical to our category. The required amount to save is how much he has saved already, $6,362.17, subtracted from the total that he needs for the car $15,998.95. :

\[ 15,998.95 - 6,362.17 = 9,636.78 \]

So, he needs to save $9,636.78 more.
Change: The final common use for subtraction that we will discuss here, is finding the change in a quantity. Whether something gets bigger or smaller, the change is the bigger value minus the smaller value.

Example 3: The percentage of young adults (age 18 to 24 years) who voted in various presidential elections are shown in the table:

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>50</td>
</tr>
<tr>
<td>1980</td>
<td>40</td>
</tr>
<tr>
<td>1984</td>
<td>41</td>
</tr>
<tr>
<td>1988</td>
<td>36</td>
</tr>
<tr>
<td>1996</td>
<td>32</td>
</tr>
<tr>
<td>2000</td>
<td>26</td>
</tr>
</tbody>
</table>

What was the change in the percentage of young adults voting between the years 1980 and 2000?

Solution: Here is a time where we use the identification numbers 1980 and 2000, to look up the values we need for the calculation. The percentage changed from 40 to 26 during that time, so the change is:

\[ 40 - 26 = 14. \]

So, the percentage changed by 14%.

Exercise 5.7.1 Answer the following problems. Use addition or subtraction as appropriate.

Difference Between:

1. Find the difference between \( 5\frac{3}{8} \) and \( 4\frac{3}{4} \).
2. Find the difference between \( 12\frac{1}{2} \) and \( 7\frac{7}{8} \).
3. Find the difference between \( 7\frac{7}{10} \) and 0.23.
4. Find the difference between $\frac{23}{100}$ and 0.09.

**Subtracted From:**

5. What is 14,375.04 subtracted from 32,191.23?

6. What is 17.8% subtracted from 55%?

**How Much More:**

7. On Tuesday, the high temperature in Pacifica was 58.7°F. By Saturday, the high temperature was 64.9°F. How much hotter was it on Saturday than on Tuesday?

8. In a math class, a student needs at least 1250 points to earn an A grade. Letty already has 987 points. How many more points does she need in order to earn her A?

9. Will was 4 feet 3 inches tall. His brother Billy was 3 feet 10 inches tall. How much taller was Will than Billy?

**Change:**

10. In 1993 the number of deaths in the U.S. due to the AIDS virus was 45,381. In 1999, the number of AIDS deaths in the U.S. was down to 16,273. What was the change in number of AIDS deaths?

11. In San Mateo County, during the 2005-2006 school year, 32.3% of 5th graders achieved 6 out of 6 fitness standards. The next year, 36.6% of 5th graders achieved 6 out of 6 fitness standards. By what percent did it change?

12. The life expectancy of a person born in the U.S. was 73.7 years in 1980, and 77.0 years in the year 2000. What was the change in life expectancy over that time?

**Mixed Problems:**

13. During the Iowa caucuses in 2008, Obama picked up 38% of the democratic delegates, Edwards got 30%, and Clinton got 29%. What percent was left over for the other candidates?

14. Jing-Jing was putting up a shelf in her closet. She needed a 2-by-4 that was $54 \frac{5}{8}$ inches long, but the lumber yard sold her a 2-by-4 that was 8 feet long. How much does she have to cut off to make the board the correct length?

15. In preparing dinner for the guests, Billy needed $2 \frac{1}{2}$ cups of chicken broth for the soup, and another $\frac{3}{4}$ cups of chicken broth for the pasta sauce. How much chicken broth does he need to prepare the dinner?

16. Paul was working the concessions stand at the basketball game. A customer ordered a popcorn for $3.50, a hotdog for $4.25, two sodas for $0.90 each, and a candy bar for $0.85. The customer gave Paul a 20 dollar bill. How much change does Paul have to give the customer?
17. If you are planning to attach a board that is \( \frac{5}{6}'' \) onto a board that is \( \frac{3}{4}'' \) thick,

(a) What is the longest screw that you can use?
(b) What is the shortest screw you can use?
(c) What is a reasonable range of screw lengths?

18. Find the difference between \( 22\frac{1}{8} \) and \( 18\frac{1}{2} \).

19. The balance in Hector’s checking account was $134.72 before the deposit. After the deposit, the balance was $473.28. How much was the deposit?

20. Geoff saws \( 13\frac{2}{3} \) inches off of a branch that measured \( 20\frac{3}{16} \) inches. How long is the branch after Geoff makes the cut?

21. Find the difference between 48% and 29%.

22. Garret wanted to try to bake bread. The recipe called for \( 4\frac{1}{2} \) cups of flour, but when he looked in the cupboard, he saw that he only had \( 2\frac{1}{2} \) cups left. How much more flour does he need?