Scientific Notation

In **scientific notation** a number is expressed as the product of a number between 1 and 10, and a power of 10. The number to the left (called the **mantissa**) is always at least 1 but less than 10. The power of 10 (called the **exponent**) is always an integer.

Examples:
57,000,000 is written $5.7 \times 10^7$
2,345,500,000,000 is written $2.3455 \times 10^{12}$
999 is written $9.99 \times 10^2$

Here are some powers of 10:

<table>
<thead>
<tr>
<th>Power</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
<th>$10^{-1}$</th>
<th>$10^0$</th>
<th>$10^1$</th>
<th>$10^2$</th>
<th>$10^3$</th>
<th>$10^4$</th>
<th>$10^5$</th>
<th>$10^6$</th>
<th>$10^7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal</td>
<td>0.001</td>
<td>0.01</td>
<td>0.1</td>
<td>1</td>
<td>10</td>
<td>100</td>
<td>1000</td>
<td>10,000</td>
<td>100,000</td>
<td>1,000,000</td>
<td>1,000,000,000</td>
</tr>
<tr>
<td>Fraction</td>
<td>$\frac{1}{1000}$</td>
<td>$\frac{1}{100}$</td>
<td>$\frac{1}{10}$</td>
<td>1</td>
<td>10</td>
<td>100</td>
<td>1000</td>
<td>10,000</td>
<td>100,000</td>
<td>1,000,000</td>
<td>1,000,000,000</td>
</tr>
</tbody>
</table>

Practice:

Rewrite these numbers in scientific notation:
2,770,000
43
91,211,000,000,000,000,000

Rewrite these numbers in decimal notation:
$3.696 \times 10^4$
$8.0 \times 10^{20}$
$1.0101 \times 10^8$
$3.96 \times 10^{-4}$

Use a calculator to simplify the following. Write your answers in scientific notation:

$\frac{3.14 \times 10^4}{2.72 \times 10^2}$

$\frac{8.76 \times 10^{-5}}{5.81 \times 10^9}$

$\frac{4.36 \times 10^3 - 1.2 \times 10^{11}}{3.01 \times 10^7}$
**Significant Digits**

When we write a number like 234,573,469,781,693,466 in scientific notation, we sometimes write $2.34 \times 10^{17}$.

This is not exactly the same as the original number because it represents 234,000,000,000,000,000.

How does rounding affect the answers in various calculations? Try these two problems:

$237 + 396$ and $237 \times 396$

You should get 633 and 93,852. If we were to round to the nearest 100, we’d have 600 and 93,900.

If instead we rounded each number in the original problem before we did the calculations, we would have $200 + 400 = 600$ and $200 \times 400 = 80,000$.

In the multiplication problem, we are off by quite a bit! The lesson here is to never round your numbers before you get the final answer. Always do as much of your computation as you can with the calculator, and round off as a last step.

*The digits that define a numerical value are called **significant digits**. For a number with no decimal part, the significant digits can be counted starting at the left and counting up to the last nonzero digit.*

Examples:

- 47,500 has 3 significant digits
- 100,200 has 4 significant digits.

Practice:

- 50,050,000 has ______ significant digits
- 23,000,000,000 has ______ significant digits

*If a number has a decimal part, the significant digits are determined by counting the total number of displayed digits, starting with the first nonzero digit on the left.*

Examples:

- 12.034 has five significant digits
- 0.00000034 has two significant digits
- 0.0000003400 has four significant digits. (We would not write the zeros at the end of the decimal unless they mean something.)

Practice:

- 57.020 has ______ significant digits
- 0.000369 has ______ significant digits
- 0.0200200 has ______ significant digits

Error Analysis

The absolute error, \( a \), is the absolute value of the difference between a measurement and the exact value.

So if a pipe that is actually 15.32 cm long and it is measured at 15.375 cm, the absolute error is
\[
a = |15.32 - 15.375| = 0.055 \text{ cm}
\]

The relative error, \( r \), is the ratio of the absolute error and the exact measurement. It is often expressed as a percent.

The relative error of a pipe that is actually 15.32 cm long and measured at 15.375 cm will be
\[
r = \frac{|15.32 - 15.375|}{15.32} = \frac{0.055}{15.32} \approx 0.0036 \text{ or } 0.36\%.
\]

Practice:
You are formatting your computer’s hard drive. You estimate the formatting time to be about 15 minutes, and it actually takes 20 minutes. Find the absolute error and the relative error.

Find the absolute error and the relative error when:
1. The exact value \( \frac{3}{7} \) is approximated with the value 0.43
2. The exact value \( \frac{5}{6} \) is approximated with the value 0.8.
3. In the U.S., a can of Coke contains exactly 355 mL of liquid. It is approximated in Australia with a can that contains 375 mL of liquid.