Section 5-1: Overview
This chapter will deal with the construction of discrete probability distributions by combining the methods of descriptive statistics presented in Chapter 2 and 3 and those of probability presented in Chapter 4.
Probability Distributions will describe what will probably happen instead of what actually did happen.

Combining Descriptive Methods and Probabilities
In this chapter we will construct probability distributions by presenting possible outcomes along with the relative frequencies we expect.

Section 5-2
Random Variables

Key Concept
This section introduces the important concept of a probability distribution, which gives the probability for each value of a variable that is determined by chance. Give consideration to distinguishing between outcomes that are likely to occur by chance and outcomes that are “unusual” in the sense they are not likely to occur by chance.

Definitions
- Random variable
  a variable (typically represented by x) that has a single numerical value, determined by chance, for each outcome of a procedure
- Probability distribution
  a description that gives the probability for each value of the random variable; often expressed in the format of a graph, table, or formula

Definitions
- Discrete random variable
  either a finite number of values or countable number of values, where “countable” refers to the fact that there might be infinitely many values, but they result from a counting process
- Continuous random variable
  infinitely many values, and those values can be associated with measurements on a continuous scale in such a way that there are no gaps or interruptions

Graphs
The probability histogram is very similar to a relative frequency histogram, but the vertical scale shows probabilities.
Requirements for Probability Distribution

\[ \sum P(x) = 1 \]
where \( x \) assumes all possible values.

\[ 0 \leq P(x) \leq 1 \]
for every individual value of \( x \).

Mean, Variance and Standard Deviation of a Probability Distribution

\[ \mu = \Sigma [x \cdot P(x)] \quad \text{Mean} \]
\[ \sigma^2 = \Sigma [(x - \mu)^2 \cdot P(x)] \quad \text{Variance} \]
\[ \sigma = \sqrt{\Sigma [x^2 \cdot P(x)] - \mu^2} \quad \text{Standard Deviation} \]

Roundoff Rule for \( \mu, \sigma, \) and \( \sigma^2 \)

Round results by carrying one more decimal place than the number of decimal places used for the random variable \( x \). If the values of \( x \) are integers, round \( \mu, \sigma, \) and \( \sigma^2 \) to one decimal place.

Identifying Unusual Results Range Rule of Thumb

According to the range rule of thumb, most values should lie within 2 standard deviations of the mean.
We can therefore identify “unusual” values by determining if they lie outside these limits:

- Maximum usual value = \( \mu + 2\sigma \)
- Minimum usual value = \( \mu - 2\sigma \)

Identifying Unusual Results Probabilities

Rare Event Rule
If, under a given assumption (such as the assumption that a coin is fair), the probability of a particular observed event (such as 992 heads in 1000 tosses of a coin) is extremely small, we conclude that the assumption is probably not correct.

- Unusually high: \( x \) successes among \( n \) trials is an unusually high number of successes if \( P(x \) or more) \( \leq 0.05 \).
- Unusually low: \( x \) successes among \( n \) trials is an unusually low number of successes if \( P(x \) or fewer) \( \leq 0.05 \).

Definition

The expected value of a discrete random variable is denoted by \( E \), and it represents the average value of the outcomes. It is obtained by finding the value of \( \Sigma [x \cdot P(x)] \).

\[ E = \Sigma [x \cdot P(x)] \]