Section 8-5
Testing a Claim About a Mean: \( \sigma \) Not Known

Key Concept
This section presents methods for testing a claim about a population mean when we do not know the value of \( \sigma \). The methods of this section use the Student t distribution introduced earlier.

Requirements for Testing Claims About a Population Mean (with \( \sigma \) Not Known)
1) The sample is a simple random sample.
2) The value of the population standard deviation \( \sigma \) is not known.
3) Either or both of these conditions is satisfied: The population is normally distributed or \( n > 30 \).

Test Statistic for Testing a Claim About a Mean (with \( \sigma \) Not Known)
\[
t = \frac{\bar{X} - \mu}{s / \sqrt{n}}
\]

P-values and Critical Values
- Found in Table A-3
- Degrees of freedom (df) = \( n - 1 \)

Important Properties of the Student t Distribution
1. The Student t distribution is different for different sample sizes (see Figure 7-5 in Section 7-4).
2. The Student t distribution has the same general bell shape as the normal distribution; its wider shape reflects the greater variability that is expected when \( s \) is used to estimate \( \sigma \).
3. The Student t distribution has a mean of \( t = 0 \) (just as the standard normal distribution has a mean of \( z = 0 \)).
4. The standard deviation of the Student t distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has \( \sigma^2 = 1 \)).
5. As the sample size \( n \) gets larger, the Student t distribution gets closer to the standard normal distribution.

Choosing between the Normal and Student t Distributions when Testing a Claim about a Population Mean \( \mu \)
Use the Student t distribution when \( \sigma \) is not known and either or both of these conditions is satisfied: The population is normally distributed or \( n > 30 \).

Example: Data Set 13 in Appendix B of the text includes weights of 13 red M&M candies randomly selected from a bag containing 465 M&Ms. The weights (in grams) have a mean \( \bar{x} = 0.8635 \) and a standard deviation \( s = 0.0576 \) g. The bag states that the net weight of the contents is 396.9 g, so the M&Ms must have a mean weight that is 396.9/465 = 0.8535 g in order to provide the amount claimed. Use the sample data with a 0.05 significance level to test the claim of a production manager that the M&Ms have a mean that is actually greater than 0.8535 g. Use the traditional method.

The sample is a simple random sample and we are not using a known value of \( \sigma \). The sample size is \( n = 13 \) and a normal quartile plot suggests the weights are normally distributed.
**Example:** Data Set 13 in Appendix B of the text includes weights of 13 red M&M candies randomly selected from a bag containing 465 M&Ms. The weights (in grams) have a mean $\mu = 0.8635$ and a standard deviation $\sigma = 0.0576$ g. The bag states that the net weight of the contents is 396.9 g, so the M&Ms must have a mean weight that is 396.9/465 = 0.8535 g in order to provide the amount claimed. Use the sample data with a 0.05 significance level to test the claim of a production manager that the M&Ms have a mean that is actually greater than 0.8535 g. Use the traditional method.

\[ H_0: \mu = 0.8535 \]
\[ H_1: \mu > 0.8535 \]
\[ \alpha = 0.05 \]
\[ \bar{x} = 0.8635 \]
\[ s = 0.0576 \]
\[ n = 13 \]

The critical value, from Table A-3, is $t = 1.782$.

**Normal Distribution Versus Student t Distribution**

The critical value in the preceding example was $t = 1.782$, but if the normal distribution were being used, the critical value would have been $z = 1.645$.

The Student $t$ critical value is larger (farther to the right), showing that with the Student $t$ distribution, the sample evidence must be more extreme before we can consider it to be significant.

**P-Value Method**

- Use software or a TI-83/84 Plus calculator.
- If technology is not available, use Table A-3 to identify a range of $P$-values.

**Example:** Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the $P$-value corresponding to the given results.

a) In a left-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -2.007$.

b) In a right-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = 1.222$.

c) In a two-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -3.456$.

**Example:** Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the $P$-value corresponding to the given results.

a) The test is a left-tailed test with test statistic $t = -2.007$, so the $P$-value is the area to the left of $-2.007$. Because of the symmetry of the $t$ distribution, that is the same as the area to the right of $+2.007$. Any test statistic between 2.201 and 1.796 has a right-tailed $P$-value that is between 0.025 and 0.05. We conclude that $0.025 < P$-value $< 0.05$. 

---

**Example:** Data Set 13 in Appendix B of the text includes weights of 13 red M&M candies randomly selected from a bag containing 465 M&Ms. The weights (in grams) have a mean $\bar{x} = 0.8635$ and a standard deviation $s = 0.0576$ g. The bag states that the net weight of the contents is 396.9 g, so the M&Ms must have a mean weight that is 396.9/465 = 0.8535 g in order to provide the amount claimed. Use the sample data with a 0.05 significance level to test the claim of a production manager that the M&Ms have a mean that is actually greater than 0.8535 g. Use the traditional method.

\[ H_0: \mu = 0.8535 \]
\[ H_1: \mu > 0.8535 \]
\[ \alpha = 0.05 \]
\[ \bar{x} = 0.8635 \]
\[ s = 0.0576 \]
\[ n = 13 \]

Because the test statistic of $t = 0.626$ does not fall in the critical region, we fail to reject $H_0$. There is not sufficient evidence to support the claim that the mean weight of the M&Ms is greater than 0.8535 g.
Example: Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the $P$-value corresponding to the given results.

b) The test is a right-tailed test with test statistic $t = 1.222$, so the $P$-value is the area to the right of 1.222. Any test statistic less than 1.363 has a right-tailed $P$-value that is greater than 0.10. We conclude that $P$-value > 0.10.

c) The test is a two-tailed test with test statistic $t = -3.456$. The $P$-value is twice the area to the right of -3.456. Any test statistic greater than 3.106 has a two-tailed $P$-value that is less than 0.01. We conclude that $P$-value < 0.01.

Requirements for Testing Claims About $\sigma$ or $\sigma^2$

1. The sample is a simple random sample.
2. The population has a normal distribution. (This is a much stricter requirement than the requirement of a normal distribution when testing claims about means.)

Section 8-6
Testing a Claim About a Standard Deviation or Variance
Key Concept

This section introduces methods for testing a claim made about a population standard deviation $\sigma$ or population variance $\sigma^2$. The methods of this section use the chi-square distribution that was first introduced in Section 7-5.

Chi-Square Distribution

Test Statistic

$$\chi^2 = \frac{(n-1) S^2}{\sigma^2}$$

- $n$ = sample size
- $S^2$ = sample variance
- $\sigma^2$ = population variance (given in null hypothesis)

$P$-Values and Critical Values for Chi-Square Distribution

- Use Table A-4.
- The degrees of freedom = $n-1$. 
Properties of Chi-Square Distribution

- All values of $\chi^2$ are nonnegative, and the distribution is not symmetric (see Figure 8-13, following).
- There is a different distribution for each number of degrees of freedom (see Figure 8-14, following).
- The critical values are found in Table A-4 using $n - 1$ degrees of freedom.

Example: For a simple random sample of adults, IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A simple random sample of 13 statistics professors yields a standard deviation of $s = 7.2$. Assume that IQ scores of statistics professors are normally distributed and use a 0.05 significance level to test the claim that $\sigma = 15$.

\[
H_0: \sigma = 15 \\
H_1: \sigma \neq 15 \\
\alpha = 0.05 \\
n = 13 \\
s = 7.2
\]

\[
\chi^2 = \frac{(n - 1)s^2}{\sigma^2} = \frac{(13 - 1)(7.2)^2}{15^2} = 2.765
\]

Properties of Chi-Square Distribution - cont

Example: For a simple random sample of adults, IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A simple random sample of 13 statistics professors yields a standard deviation of $s = 7.2$. Assume that IQ scores of statistics professors are normally distributed and use a 0.05 significance level to test the claim that $\sigma = 15$.

\[
H_0: \sigma = 15 \\
H_1: \sigma \neq 15 \\
\alpha = 0.05 \\
n = 13 \\
s = 7.2
\]

\[
\chi^2 = 2.765
\]

Because the test statistic is in the critical region, we reject the null hypothesis. There is sufficient evidence to warrant rejection of the claim that the standard deviation is equal to 15.

The critical values of 4.404 and 23.337 are found in Table A-4, in the 12th row (degrees of freedom = $n - 1$) in the column corresponding to 0.975 and 0.025.