5.1 - Introduction to Polynomials and Polynomial Functions

1. **Vocabulary of Polynomials:**
   (a) **Polynomial:** (translates to “many terms”) A polynomial is the sum of (or difference of) many (one or more) terms whose variables have non-negative integer exponents.

   (b) **Term:** An algebraic term is an expression that contains constants and/or variables. The terms of a polynomial are the “things” separated by the plus/minus signs.

   (c) **Coefficient:** The numerical coefficient of a term is the number “attached” to that term.

   (d) **Monomial:** A simplified polynomial that has exactly one term is called a ______________.

   (e) **Binomial:** A simplified polynomial that has exactly _____ terms is called a binomial.

   (f) **Trinomial:** A simplified polynomial that has exactly _____ terms is called a trinomial.

   We can make up an example of a polynomial:

   What are the coefficients of our example?

   **Example 1:** Circe the following which are polynomials “in x.”

   \[
   4x^2 \quad \sqrt{2}x^3 - 8.7x - 2 \quad \frac{4}{x - 2} + 10 \\
   x^\frac{1}{3} - 4x^2 \quad x^{-3} + 10 \quad 9x^9 - 7x^{12} + 4x
   \]

   (g) **Degree:** The degree of a monomial \( ax^n \) is \( n \). For example, what is the degree of \( 5x^{12} \)? ____

   (h) **Standard Form:** A polynomial which is written with the terms in order of descending powers of the variable is written in standard form.

   **Example 2:** Write the polynomial \( 7x^2 - 6.5x^8 + 9x^4 + 3 - x \) in standard form.

   (i) **Polynomial in many variables:** A polynomial in many variables is the sum of one or more terms of the form \( ax^n y^m z^k \) (could have more variables!) with \( a \in \mathbb{R} \) and \( n, m, k \) non-negative integers. The degree of \( ax^n y^m z^k \) is \( n + m + k \).
(j) **Degree of a Polynomial**: The degree of a polynomial is the same as the degree of the term in the polynomial with the largest degree.

(k) **Leading Term**: The term of the polynomial with greatest degree is called the leading term (when we write the polynomial in standard form, this term comes first).

(l) **Leading Coefficient**: The coefficient of the leading term is called the leading coefficient.

**Example 2**: Consider the polynomial $21 + 3x^2 - 5x^9 + 12x^{10} + 4x$.

(i) Identify the coefficient and degree of each term.

$$21 + 3x^2 - 5x^9 + 12x^{10} + 4x$$

(ii) Write the polynomial in standard form:

(iii) The leading term is ________.

(iv) The leading coefficient is ________.

(v) The degree of the polynomial is ________.

2. **Polynomial Functions**: We modeled many things in chapters 1 - 5 with linear functions. In reality, we can model many situations more accurately using polynomial functions. In a polynomial function, the expression that defines the function is a polynomial.

**Example 2**: Let $P(x) = 2x^2 - 3x + 1$. Find:

(a) $P(0)$  (b) $P(-1)$  (c) $P(m)$  (d) $P(-3t)$.

**Example 3**: The height of a ball (in feet) kicked into the air is modeled by the polynomial function $h(t) = -16t^2 + 58t$, where $t$ is seconds after being kicked. Find each of the following and interpret your answer using values and appropriate units: $h(0), h(1),$ and $h(3.6)$.
3. **Graphs of Polynomials:**
Fact: the graphs of polynomial functions are “smooth and continuous.” This simply means that we can draw them without lifting up our pencils, and they don’t have any “spikes.”

Graphs of polynomials:  
Not graphs of polynomials:

4. **End Behavior of Polynomial Functions:** The end behavior of a polynomial function is referring to what the polynomial does as we plug in large positive $x$-values and large negative $x$-values. In other words, what the polynomial does to the “far right and far left.”

**Example 4:** Below is the graph of a polynomial. Describe the end behavior.

![Graph of a polynomial]

The end behavior is dependent upon the **leading term**, both the coefficient of the leading term and the degree of the leading term. Your book calls this the “leading coefficient test.” It can be summarized by the following table. For each, the leading term is $ax^n$:

<table>
<thead>
<tr>
<th>$a$ positive</th>
<th>$a$ negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ odd</td>
<td>$n$ odd</td>
</tr>
</tbody>
</table>

Ex.:  
Ex.:  
Ex.:  
Ex.:
**Example 5:** Describe and draw the end behavior of each polynomial function:

(a) \( f(x) = -5x^4 + 2x^3 + 5 \)  
(b) \( f(x) = 8x^5 + x^2 + 5x - 2 \)

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5. **Adding and Subtracting Polynomials (review):**
We can add and subtract polynomials by combining like terms. When subtracting polynomials, do not forget to distribute the negative sign!

**Example 6:** Add the polynomials

(a) \( (xy^2 - 2x^2 + 7x^2y) + (-4xy^2 - y^2 + 2x^2y) \)  
(b) \( (4y^{2n} + 2y^n - 1) + (-5y^{2n} + y^n - 8) \)

**Example 7:** Subtract the polynomials

(a) \( (3x^3 - 2x + 1) - (8x^3 + 5x - 9) \)  
(b) \( (5c^2d + 3cd) - (7c^2d + 5cd) \)