Practice Set 6.1
Rational Expressions and Functions: Multiplying and Dividing

In exercises 1-6, use the given rational function to find the indicated function values. If a function does not exist, so state.

\[ f(x) = \frac{x^2}{x - 5} \quad \quad g(x) = \frac{t^3 + 1}{t^2 - 9} \]

1. \( f(4) \) 1. _____________
2. \( g(1) \) 2. _____________
3. \( f(5) \) 3. _____________
4. \( g(0) \) 4. _____________
5. \( f(0) \) 5. _____________
6. \( g(3) \) 6. _____________

In exercises 7-9, find the domain of the given rational function. Use set builder notation.

7. \( f(x) = \frac{x+1}{x-6} \) 7. _____________
8. \( f(x) = \frac{x-4}{(x+2)(x-1)} \) 8. _____________
9. \( f(x) = \frac{x+9}{x^2 + 3x + 2} \) 9. _____________

In exercises 10-20, complete the problem by (a) factoring first and then (b) simplifying to lowest terms.

10. \( \frac{x^2 - 25}{x^2 - 9} \cdot \frac{x + 3}{x + 5} \) 10a. _____________
    b. _____________
11. \( \frac{5x - 10}{x - 2} \) 11a. _____________
    b. _____________
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12. \( \frac{a^3 - 9a}{a^2 - 3a} \cdot \frac{a - 1}{a^2 - 3a} \)
   
   12a. ______________
   b. ______________

13. \( \frac{3x - 12}{x^2 - 4} \cdot \frac{x^2 + 6x + 8}{x - 4} \)
   
   13a. ______________
   b. ______________

14. \( \frac{2x^2 - 5x - 12}{4x^2 + 8x + 3} \div \frac{x^2 - 16}{2x^2 + 7x + 3} \)
   
   14a. ______________
   b. ______________

15. \( \frac{3 - x}{x^2 - 9} \)
   
   15a. ______________
   b. ______________

16. \( \frac{x^2 + 3x + 2}{x + 5} \div x + 1 \)
   
   16a. ______________
   b. ______________

17. \( \frac{x^2 - 2x + 1}{3x^2 + 7x - 20} \div \frac{x^2 + 3x - 4}{3x^2 - 2x - 5} \)
   
   17a. ______________
   b. ______________

18. \( \frac{x^2 - 5x + 6}{x^2 - 2x - 3} \cdot \frac{2x^2 - 2}{2x^2 - 4x} \)
   
   18a. ______________
   b. ______________

19. \( \frac{2x^2 - 3x - 20}{x - 4} \)
   
   19a. ______________
   b. ______________

20. \( \frac{x + 2}{x^2 - 5x + 6} \div \frac{x^2 - 3x - 10}{x^2 + 2x - 8} \)
   
   20a. ______________
   b. ______________
Practice Set 6.2
Adding and Subtracting Rational Expressions

Add or subtract. Note in each problem the denominators are the same. Simplify the result if possible.

1. \( \frac{3}{5x} + \frac{1}{5x} \)

2. \( \frac{x}{x - 2} - \frac{4x - 3}{x - 2} \)

3. \( \frac{x}{x^2 - 2x - 3} + \frac{1}{x^2 - 2x - 3} \)

4. \( \frac{3x + 5}{x - 1} - \frac{2x - 4}{x - 1} \)

5. \( \frac{2x - 2}{x - 6} - \frac{x + 4}{x - 6} \)

6. \( \frac{x^2 + 2x + 9}{x^2 + 7x + 10} - \frac{-x + 7}{x^2 + 7x + 10} \)

Find the least common denominator of the rational expressions.

7. \( \frac{9}{15x^2} \) and \( \frac{4}{12x^4} \)

8. \( \frac{1}{x + 4} \) and \( \frac{6}{x^2 - 16} \)

9. \( \frac{3}{x(x + 2)} \) and \( \frac{7}{x^2 - 4} \)

10. \( \frac{8}{x^2 - 2x + 1} \) and \( \frac{x}{(x^2 - 1)} \)

11. \( \frac{9x}{4x - 8} \) and \( \frac{3x + 1}{x^2 + 2x - 8} \)

12. \( \frac{5x}{x^2 + 9x + 20} \) and \( \frac{x + 7}{x^2 + x - 12} \)
Add or subtract. Note in each exercise the denominators are different.

13. \( \frac{7}{3x^2} + \frac{9}{4x} \)

14. \( \frac{8}{x-4} - \frac{3}{x + 2} \)

15. \( \frac{1}{x-y} + \frac{1}{y-x} \)

16. \( \frac{3x+1}{2x-6} - \frac{x+2}{x-3} \)

17. \( \frac{1}{a+b} + \frac{3ab}{a^3 + b^3} \)

18. \( \frac{8}{2x^2 + x - 6} - \frac{2}{3x^2 + 4x - 4} \)

19. \( \frac{2}{x^2 + 5x + 6} - \frac{4}{x^2 + 4x + 3} \)

20. \( \frac{4x}{x^2 + 6x + 5} - \frac{3x}{x^2 + 5x + 4} \)
Practice Set 6.3
Complex Rational Expressions

(a) State the LCD (least common denominator) for each of the following complex rational expressions, then (b) simplify each complex rational expression.

1. \( \frac{x - 2}{4} - \frac{1}{3 + \frac{1}{x}} \)
   1a. _____________
   1b. _____________

2. \( \frac{3}{x^2 + \frac{1}{x}} + \frac{2}{2 - \frac{4}{5x}} \)
   2a. _____________
   2b. _____________

3. \( \frac{1}{x} + \frac{1}{y} + \frac{1}{\frac{1}{x} - \frac{1}{y}} \)
   3a. _____________
   3b. _____________

4. \( \frac{3}{x + 2} - \frac{3}{x - 2} \)
   4a. _____________
   4b. _____________

5. \( \frac{1}{y} - \frac{3}{2 + \frac{4}{y}} \)
   5a. _____________
   5b. _____________
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6. \[
\frac{4x^2y^2 - 1x^{-1}y^{-2}}{16x^{-2}y^{-2} - y^{-2}}
\]

6a. _____________

6b. _____________

7. \[
\frac{2}{x+2} \div \frac{1}{x+2} + \frac{2}{x}
\]

7a. _____________

7b. _____________

8. \[
\frac{4 - \frac{1}{x^2}}{4 + \frac{4}{x} + \frac{1}{x^2}}
\]

8a. _____________

8b. _____________

9. \[
\frac{\frac{5}{1 + \frac{x + 4}{2}}}{\frac{2}{10}} - 3
\]

9a. _____________

9b. _____________

10. \[
\frac{\frac{2x}{x + 2}}{\frac{x}{x + 2} + \frac{x}{x + 2}}
\]

10a. _____________

10b. _____________
Practice Set 6.4
Division of Polynomials

Divide the polynomial by the monomial.

1. \[
\frac{35x^6 - 20x^4 + 5x^2}{5x^2}
\]
2. \[
\frac{49x^5 - 7x^3 + 14x}{7x^4}
\]
3. \[
\frac{12x^8 - 16x^4 + 14x^2}{2x^3}
\]
4. \[
\frac{21x^9 + 7x^5 - 28x}{-7x^5}
\]
5. \[
\frac{9a^2b^2 + 15a^3b^3 + 18a^4b^4}{3a^2b^2}
\]
6. \[
\frac{4a^3b^5 - 8a^2b^6 - 12ab^7}{-4ab^5}
\]
7. \[
\frac{6x^2 y^2 + 4xy + 2}{2xy}
\]
8. \[
\frac{24x^3 y - 16x^2 y^2 + 18xy^3 - 20y^4}{-2y}
\]
9. \[
\frac{11a^4b^3 + 22a^3b^4 - 55ab^5}{11ab^3}
\]
10. \[
\frac{50x^2 y^2 + 30x^3 y^3 - 40x^4 y^4}{-10x^2 y^3}
\]
Divide as indicated.

11. \( (x^2 - x - 6) \div (x + 2) \)

12. \( (x^2 + 5x + 6) \div (x + 3) \)

13. \( (x^3 - 125) \div (x - 5) \)

14. \( (4x^2 - 15x + 6) \div (x - 3) \)

15. \( (x^2 + 4x - 5) \div (x - 3) \)

16. \( (3x^3 - 7x^2 - 5x + 11) \div (x - 3) \)

17. \( (2x^3 + 3x^2 - 6x - 3) \div (x + 2) \)

18. \( (2x^3 + 7x^2 + 4x + 6) \div (x + 2) \)

19. \( (3x^3 - 11x^2 + 7x + 1) \div (x - 3) \)

20. \( (x^4 + x^3 - 1) \div (x + 2) \)
Use synthetic division to divide:

1. \((x^2 + 3x + 2) \div (x + 2)\)
2. \((2x^2 - x - 3) \div (x + 1)\)
3. \((x^2 + 7x - 2) \div (x - 2)\)
4. \((3x^2 + 9x + 1) \div (x + 3)\)
5. \((2x^3 - 3x^2 - 4x + 5) \div (x + 1)\)
6. \((6y^3 - 8y + 5) \div (y - 2)\)
7. \((x^4 - 2x^3 - 70x + 20) \div (x - 5)\)
8. \((x^4 + x^3 - 3x^2 - x + 2) \div (x + 2)\)

Use synthetic division and the Remainder Theorem to find the indicated function value.

9. \(f(x) = x^3 + 2x^2 - x + 4; f(1)\)
10. \(f(x) = 2x^3 - 3x^2 + 4x - 7; f(-2)\)
11. \(f(x) = 3x^3 + 5x^2 - 2x + 1; f(-3)\)
12. \(f(x) = 4x^3 - 3x^2 - 2x - 1; f(-4)\)
13. \(f(x) = 2x^4 + 3x^3 + 5x^2 - 6x + 9; f(-2)\)
14. \(f(x) = 3x^4 - 2x^3 + 6x^2 + 7x - 3; f\left(\frac{1}{2}\right)\)
Use synthetic division to show that the number given to the right of each equation is a solution of the equation. Then solve the polynomial equation.

15. \(3x^3 + 5x^2 - 47x + 15 = 0\); 3
16. \(2x^3 + 3x^2 - 18x + 8 = 0\); 2
17. \(2x^3 - 3x^2 - 18x - 8 = 0\); 4
18. \(3x^3 - 5x^2 - 11x - 3 = 0\); -1
19. \(6x^3 + 29x^2 - 6x - 5 = 0\); -5
20. \(2x^3 - 11x^2 + 10x + 8 = 0\); 2
**Practice Set 6.6**

Rational Equations

Solve each rational equation. If an equation has no solution, so state.

1. \[
\frac{5}{x} + \frac{5}{2x} = 15
\]

2. \[
\frac{1}{x} + 3 = \frac{4}{x}
\]

3. \[
\frac{2}{x+5} = \frac{2}{5}
\]

4. \[
\frac{5}{x+3} = \frac{6}{8}
\]

5. \[
\frac{x+4}{x^2+5x} = \frac{-2}{x^2-25}
\]

6. \[
1 - \frac{1}{x} = \frac{12}{x^2}
\]

7. \[
\frac{x}{6} = \frac{5}{x-1}
\]

8. \[
\frac{x}{x+1} - \frac{1}{2} = \frac{-1}{x+1}
\]

9. \[
\frac{x+1}{3x+12} = \frac{8}{x^2-16}
\]

10. \[
\frac{x-4}{x^2-3x} = \frac{-2}{x^2-9}
\]
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11. \[ \frac{1}{x-1} - \frac{1}{x+1} = \frac{2x}{x^2 - 1} \]  
   11. _______________

12. \[ \frac{6}{x^2} = \frac{5}{x} \]  
   12. _______________

13. \[ \frac{x+3}{10} - \frac{x-4}{15} = \frac{x+2}{20} \]  
   13. _______________

14. \[ \frac{3}{x^2} - \frac{4}{x} = -1 \]  
   14. _______________

15. \[ \frac{7}{x^2 - 4x + 3} = \frac{1}{x-1} + \frac{2}{x-3} \]  
   15. _______________

16. \[ \frac{2}{x^2 + 4x + 3} = \frac{-1}{x+3} \]  
   16. _______________

17. \[ \frac{5}{x+5} - 3 = \frac{9}{x+5} \]  
   17. _______________

18. \[ \frac{4x}{x+2} - \frac{x}{x-1} = \frac{9}{x^2 + x - 2} \]  
   18. _______________

19. \[ \frac{2}{x+1} + \frac{3}{x-3} = \frac{2}{x^2 - 2x - 3} \]  
   19. _______________

20. \[ \frac{1}{x-1} = \frac{3}{x+2} - \frac{5}{x^2 + x - 2} \]  
   20. _______________
Solve each formula for the specified variable.

1. \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \) for \( p \)

2. \( \frac{V_1}{V_2} = \frac{P_2}{P_1} \) for \( V_1 \)

3. \( A = \frac{1}{2} (b_1 + b_2)h \) for \( b_1 \)

4. \( A = \frac{1}{2}bh \) for \( b \)

5. \( P = \frac{A}{1 + rt} \) for \( t \)

6. \( z = \frac{x - \bar{x}}{s} \) for \( \bar{x} \)

7. \( F = \frac{9}{5}C + 32 \) for \( C \)

8. \( V = \frac{1}{3} \pi r^2h \) for \( h \)
9. The current of a river is 8 mph. A small boat can travel 30 miles against the current in the same amount of time it can travels 20 miles with the current. Find the speed of the boat in still water.

Step 1: Let \( x = \) ________________

Step 2: Represent the rates in terms of \( x \)
- with the current _____________
- against the current ___________

Step 3:

<table>
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<th></th>
<th>Distance</th>
<th>Rate</th>
<th>Time = Distance/Rate</th>
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<td>with the current</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>against the current</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 4: Equation: __________________________________________________________

Solve the equation and answer the question.

Step 5: Check

10. Bill can mow 5 lawns in 7 hours. Jake can mow the same 5 lawns in 8 hours. Find out how long it would take Bill and Jake to mow the lawns if they worked together.

Step 1: _______________ time it takes Bill alone

_______________ time it takes Jake alone

Step 2: _______________ = time it takes for Bill and Jake to complete the job together

Step 3:

<table>
<thead>
<tr>
<th></th>
<th>Fractional part of job done in 1 hour</th>
<th>Time working together</th>
<th>Fractional part of job completed in 1 hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bill</td>
<td></td>
<td>( x )</td>
<td></td>
</tr>
<tr>
<td>Jake</td>
<td></td>
<td>( x )</td>
<td></td>
</tr>
</tbody>
</table>

Equation: __________________________________________________________

Step 4: Solve the equation and answer the question.

Step 5: Check
Practice Set 6.8
Modeling Using Variation

Use direct, inverse, joint or combined variation to solve each problem.

1. \( y \) varies directly with the cube root of \( x \). If \( y \) is 14 when \( x \) is 8, find \( y \) when \( x \) is 64.

2. \( y \) varies inversely with the square root of \( x \). If \( y \) is 10 when \( x \) is 36, find \( x \) when \( y \) is 12.

3. \( m \) varies jointly with \( x \) and the square of \( y \). If \( m \) is 54 when \( x \) is 3 and \( y \) is 3, find \( m \) when \( x \) is 2 and \( y \) is 4.

4. \( y \) varies directly with \( x \) and inversely with \( p \). If \( y \) is 9 when \( x \) is 12 and \( p \) is 4, find \( y \) if \( x \) is 30 and \( p \) is 5.

5. \( x \) varies directly as \( y \). If \( x = 4 \) and \( y = 2 \), find \( x \) when \( y = 10 \).

6. \( t \) varies directly with \( x \) and \( y \). If \( t \) is 25 when \( x \) is 10 and \( y \) is 5, find \( t \) when \( x \) is 12 and \( y \) is 4.

7. Property tax “\( t \)” varies directly to the assessed value of a home. If a house valued at $110,000 is taxed $2,750, what would be the tax on a house valued at $85,000?

8. Joe Angelo earns a commission on each appliance he sells. If his commission varies directly to the amount of sales. If he sells a dishwasher for $300 and earns a commission of $36, how much commission will he earn if he sells a plasma television for $800?

9. The volume of gas varies inversely to the pressure. If the pressure of 36 pounds per square inch corresponds to a volume of 25 ft.\(^3\), what pressure is needed to produce a volume of 75 ft.\(^3\)?

10. The capacity of a cylinder varies jointly with the height and square of the radius. If a cylinder has a capacity of 3 m\(^3\) when the radius is 3 meters and the height is 6 meters, what will the capacity of a cylinder be if the radius is 4 meters and the height
is 9 meters?