Problem #1:
(Note: This is exactly the same problem you considered in Friday’s “Sanity Check” quiz – but now I would like you to derive the answer directly, using electric forces and Newton’s Laws. Do you get the same answer you chose on Friday’s quiz?)

Two square metal plates of side length $L$ are separated by a distance $z$ (where $z \ll L$). The space between the plates is filled with a uniform electric field $E$, pointing downward as shown in the diagram below.

A particle with mass $m$ and charge $-q$ is launched from a point midway between the plates, with a (perfectly horizontal) initial velocity $v_0$. The particle curves upward and strikes the top plate a horizontal distance $x$ from its launch point.

You may neglect gravity and all forms of friction or air resistance.

What is the horizontal distance $x$ that the particle travels before striking the top plate?

(Hint: This is pretty much exactly the same problem as one that will be quite familiar from Physics 4A (or any equivalent class): a projectile launched horizontally in a uniform gravitational field.)

Problem #2:
In lecture, we found that the electric field of a uniformly charged disc, measured at a point on the disc’s axis, at distance $r$ from the disc, is given by:

$$E = 2\pi k_e \sigma \left(1 - \frac{r}{\sqrt{r^2 + R^2}}\right)$$

where $R$ is the disc’s radius, and $\sigma$ its charge per unit area ($\sigma = Q / A$).
At a great enough distance, the disc ought to “look” like a point charge. Show that it indeed does. In other words, show that in the limit $r \gg R$, the above equation reduces to the Coulomb equation for the field of a point charge,

$$E = k_c \frac{Q}{r^2}$$

Hint: If $r \gg R$, then the ratio $(R / r)$ is a small, dimensionless quantity. Try a power series expansion in powers of this small quantity.

**Problem #3:**

A helium nucleus consists of two protons and two neutrons, packed into a tiny region of space. (The average center-to-center separation of neighboring particles in a nucleus is around 1 fm = $10^{-15}$ m, about five orders of magnitude smaller than the size of the electron cloud which makes up the rest of the atom.)

*For the purposes of this problem, you may assume the protons are separated (center-to-center) by exactly 1 fm.*

a) Why might it seem strange that the nucleus can exist at all? What would the electrical force “want” those two protons to do?

b) Your dog, who takes a keen interest in particle physics, suggests that the nucleus is held together by gravity. The dog has a point: there is a gravitational force, pulling those particles toward one another.

Compare the gravitational attraction between those two protons to the electrical repulsion. Which force is stronger? How *much* stronger? Could the dog be correct that gravity holds the nucleus together?

c) The force which actually holds those protons in place is the Strong Nuclear Force (SNF). Suppose the SNF were suddenly to vanish. Calculate the instantaneous acceleration that one of those two protons in the helium nucleus would suddenly undergo.

d) If the proton continues to accelerate at the rate you calculated in part (c), how long would it take the proton to reach the speed of light ($c = 3 \times 10^8$ m/s)? How much distance would the proton move before reaching that speed?

e) If the SNF in helium really did vanish, would that proton actually reach the speed of light? Why or why not? (There are many reasons you could cite – can you give at least two?)

**Problem #4:**

You have investigated the properties of an electric dipole, which is essentially a “lopsided” charge distribution, with more positive charge on one side, and more negative charge on the other. (The *simplest* example of a dipole is two point charges separated by a tiny distance, but *any* lopsided charge distribution will behave as a dipole, when observed from a great distance.)
The following charge distribution is not lopsided; its dipole moment is zero. It is, in fact, one simple representation of an electric quadrupole.

```
<------- a ------> <------- a ------>
o          o
-2q         +2q
<--------------------- r --------------------->
```

Three point charges are arranged along a line, with equal separation $a$. The middle charge is +2q; the two outside charges are -q. We want to calculate the electric field at a point along the axis of this distribution, at a distance $r$ from the central charge.

a) Write an exact expression for the E-field at point P, using Coulomb’s Law and field superposition.

b) If point P is far from the charge distribution – in other words, if $r >> a$ – then you can write an approximate, but much simpler, expression for the E-field. Expand your result from part (a) in a power series in the small quantity ($a / r$) – much like we did in class, for the case of a dipole.

Continue including terms in your power series, until the first term that does not vanish. (We often talk about expanding a power series to its first non-vanishing term.)

c) What you have found in part (b) is the (on-axis) electric field of an ideal electric quadrupole. What would you say is its most important difference from the fields of an electric monopole (a fancy name for a point charge) and/or an electric dipole (which we derived in class?)

d) If I had made the central charge +5q instead of +2q, and left all other aspects of the problem the same what would have been the first non-vanishing term in your power series from part (b)? Can you give a brief explanation of why that result makes sense?

**Problem #5:**

Air is normally a poor conductor of electrical current. However, as you have probably seen, a strong enough electric field can cause the air to “break down” and become a conductor. (This is how lightning works, from tiny static sparks to dramatic thunderbolts.)

The threshold field required to convert air from an insulator to a conductor is:

$$E_{\text{breakdown}} = 3 \times 10^6 \text{ N/C}.$$

a) What would be the minimum amount of charge that must be placed on a conducting sphere of radius $R$, in order for it to “break down” the air around it and start throwing off electric sparks?

b) What would be the minimum linear charge density (charge per unit length) that must be placed on a conducting cylinder of radius $R$, in order for it to “break down” the air around it and start throwing off electric sparks?
c) What would be the minimum area charge density (charge per unit area) that must be placed on a flat conducting surface, in order for it to “break down” the air around it and start throwing off electric sparks?

Problem #6:
A thin, flat vertical sheet (essentially a “wall”) carries uniform charge density $\sigma = 2 \times 10^{-6}$ C/m$^2$. The diagram to the right shows an “edge-on” view of the wall, which extends for a great distance, both vertically and out of the page.

A small sphere of mass $m = 0.25$ g is suspended from a string of length $L = 0.20$ m, tied to a point on the wall as shown.

When this whole setup reaches equilibrium, the string makes an angle $\theta = 30^\circ$ with the thin sheet.

a) Calculate the charge $q$ on the small sphere.

b) Would you characterize this equilibrium as stable, unstable, or neither? Briefly explain your reasoning.

c) Which of the given parameters was not needed in order to calculate the answer? Were you surprised that parameter was not needed? Why or why not?

d) Calculate the tension in the string.

e) (BONUS SECTION) Calculate the frequency of small oscillations about the equilibrium position shown.

(Hint: You don’t need any calculus or power series expansions for this question...but it might take some creativity and lateral thinking to connect this situation to the oscillators you learned about in Physics 4A or the equivalent. Thinking about similar situations you’ve seen before, and how your answers to part (c) and (d) would relate to, and/or modify, those situations, might be a good start to your brainstorming.)
Problem #7: Consider a spherical cow…

Work out a Fermi (order-of-magnitude) estimate of the amount of charge you could place on a cow, before the cow began discharging sparks into the air around it.

As always on a Fermi estimate, don’t let yourself get bogged down in details of bovine physiology that will delay your thought process. Any time you feel you need detailed information that’s not easily available, make your best estimate, justify why you think that estimate is probably good to at least the nearest order of magnitude, and move on with the calculation...

According to the Internet, this is “Dr. Mike and the spherical cow of uniform density.”
The cow was made by Irene Raun.
I haven’t been able to figure out at which event this picture was taken – bragging rights to anyone who can!