Chapter 1: Logic of Compound Statements

First: Aristotle (Gr. 384-322 BC)
Collection of rules for deductive reasoning to be used in every branch of knowledge

Next: Gottfried Leibniz (German, 17th century) using symbols to mechanize the process of logic as algebraic notation had mechanized the process of reasoning about numbers and their relationships.

Next: George Boole and Augustus De Morgan (English, 18th century) founded modern logic.

1.1 Logical Form and Logical Equivalence
Concept of argument form is central to the concept of deductive logic. An argument is a sequence of statements aimed at demonstrating the truth of an assertion.

THE FORM OF AN ARGUMENT IS DISTINGUISHED FROM ITS CONTENT
So, logic won't determine the merit of argument's content; it can only determine if the conclusion follows from the truth of preceding statements.

LOGICAL FORM: The following two arguments have the same logical form but differ entirely in content.

If the program syntax is faulty or if program execution results in division by zero, then the computer will generate an error message. Therefore, if the computer does not generate an error message, then the program syntax is correct and the program execution does not result in division by zero.

If $x$ is a real number such that $x < -2$ or $x > 2$, then $x^2 > 4$. Therefore, if $x^2 \leq 4$, then $x \geq -2$ and $x \leq 2$.

Using $p$, $q$, $r$, to represent the three statements in each argument.

If $p \text{ or } q$, then $r$
Therefore, if $\neg r$, then $\neg p$ and $\neg q$
STATEMENT:
A statement is a sentence that is true or false but not both.

Self-referential sentences--The Barber Puzzle

**In a certain town there is a male barber who shaves all those men, and only those men, who do not shave themselves. Question: Does the barber shave himself?**

**COMPOUND STATEMENTS**

- negation of p \[ NOT (\sim) \]
- conjunction of p \[ AND (\wedge) \]
- disjunction of p \[ OR (\vee) \]

**TRANSLATING ENGLISH INTO SYMBOLS**

**But =** AND

*It is not sunny but it is hot.*

\[ \sim p \wedge q \]

**Both and =** it is not the case that the sentence _______

*It is not both sunny and hot.*

\[ \sim(p \wedge q) \]

**Neither/nor =** not A and not B

*It is neither hot nor sunny.*

\[ \sim p \wedge \sim q \]

**LOGIC**

**PROPOSITIONS AND CONNECTIVES**

**Proposition (Statement) is a declarative sentences that is either true or false.**

- Tetanus is a disease.
- \( \frac{1}{2} \) is an integer.
- There is intelligent life on Mars.
One plus one equals two.

We denote propositions by **lowercase letters** p, q, r, s, etc., called propositional variables.

The **truth values** of a proposition are either TRUE or FALSE.

**CONNECTIVES**

**Negation (NOT)**

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**Conjunction (AND)**

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**Disjunction (OR)**

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1.2 Conditional Statements

Implication

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- p implies q only if q is p
- if p then q q is necessary for p
- q if p p is sufficient for q
- p only if q

IMPLICATION AND VACUOUS TRUTH

**Example:** If you loan me $20.00 then I will pay you on Friday.

p = you loan me $20.00 q = I will pay you on Friday

**QUESTION:** Under what conditions can you call me a liar for not keeping my word?

You loan me $20.00 (p is true), Friday I don't pay you (q is false)

What about the other conditions?

You loan me $20.00 (p is true) Friday I pay you (q is true) *No problem here.*

You don't loan me $20.00 (p is false) Friday I pay you (q is true) *Crazy maybe but not a liar.*

You don't loan me $20.00 (p is false) Friday I don't pay you (q is false) *No problem here.*

Because there is really only one condition which would make the conditional
statement false (I'd be a liar) the remaining conditions are said to be vacuously true.

**ORDER OF OPERATIONS**

negation
conjunction (left to right)
disjunction (left to right)
implication
equivalence

**CONVERSE, CONTRAPOSITIVE, AND INVERSE OF A PROPOSITION**

Prove that these are logically equivalent.
- If the proposition is \( p \rightarrow q \), then its contrapositive is \( \neg q \rightarrow \neg p \)

Prove that these are logically equivalent.
- If the proposition is \( p \rightarrow q \), then its inverse is \( \neg p \rightarrow \neg q \)
- If the proposition is \( p \rightarrow q \), then its converse is \( q \rightarrow p \)

**DEMORGAN'S LAWS**

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TAUTOLOGIES --> always true conclusion

CONTRADICTIONS --> always false conclusion

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<th>Tautology</th>
<th>Contradiction</th>
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LOGICAL EQUIVALENCE  (Table pg. 14)

Logical Equivalence for implication

if p then q    \( p \Rightarrow q \)    \( \equiv \neg p \lor q \)

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THE NEGATION of and Implication

if p then q    \( \neg (p\rightarrow q) \)

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IFF=IF AND ONLY IF IS A BICONDITIONAL

PROVE: p ⇔ q ≡ (p → q) ∧ (q → p)

A LOGIC REMINDER: a hypothesis and conclusion are not required to have related subject matter.

1.3 VALID AND INVALID ARGUMENTS

- An argument is a sequence of statements.
- All but the final statement are called premises.
- The last statement is called the conclusion and is usually preceded by ∴.

A valid argument means the FORM is valid.

First we locate the premises and the conclusion(s).
Next we create an appropriate Truth Table.
Find the critical rows. All premises are T, then the conclusion(s) must be true.

We will confirm that the following valid arguments:

**MODUS PONENS** (method of affirming)

p → q
p
∴ q

**MODUS TOLLENS** (method of denying)

p → q
¬q
∴ ¬p

**DISJUNCTIVE SYLLOGISM**

p ∨ q
¬q
∴ p
∴ q

**HYPOTHETICAL SYLLOGISM** (or transitive property of implication)

p → q
FALLACIES

- Vague or ambiguous premises
- Begging the question (assuming what is to be proven)
- Jumping to a conclusion without grounds
- Converse error
- Inverse error

Converse Error and Inverse Error
These look like MODUS PONENS & MODUS TOLLENS

CONTRADICTION RULE: the heart of proof by contradiction
If an assumption leads to a contradiction, then that assumption must be false.

\[ \neg p \rightarrow c \]
\[ \therefore c \]

Knights and Knaves (Raymond Smullyan, pg. 39-40)
Knights always tell the truth.
Knaves always lie.
A says: B is a knight.
B says: A and I are opposite types.

What are A and B?

Okay, assume A is a knight. (always tells the truth)

A says: B is a knight.
B says: A and I are opposite types.

Contradiction. They cannot both be knights and be of opposite types.

Okay, assume A is a knave (always lies).

A says: B is a knight.
B says: A and I are opposite types.

Not telling the truth, so they aren’t of opposite types.

They are both Knaves.

Summary of Rules of Inference (pg. 40-Valid Argument Forms)

1.4 DIGITAL LOGIC GATES (tutorial videos here is a link to Simple Circuits) and Boolean Algebra
Boolean Expressions corresponding to Circuits

1.5 NUMBER SYSTEMS AND CIRCUIT ADDITION
Subtraction (we will not cover Circuits for Computer Addition 61-63)

Two’s Complement and the Computer Representation of Negative Integers

Computer Addition with Negative Integers

Hexadecimal Notation-Conversions

Representation of Floating Point Numbers: IEEE