Twelve Rings/Five Fingers

You have 5 fingers and a bucket full of 12 beautiful golden rings. How many ways can you place the rings on your fingers?

Here's a way of thinking about the problem:
0) How many ways can you put 0 rings on 5 fingers? ANSWER: 1
1) How many ways can you put 1 ring on 5 fingers? ANSWER: 5 (I know....)
PLUS
2) How many ways can you put 2 rings on 5 fingers? ANSWER: 5 stacked and 10 unstacked
PLUS
3) How many ways can you put 3 rings on 5 fingers? ANSWER: 5 stacked(3) and ...
PLUS
PLUS
PLUS
12) How many ways can you put 12 rings on 5 fingers?

Include the answers to the following questions as part of your solution write-up for this problem. Please word process your solution.
1. How does this problem relate to Pascal's Triangle?
2. How does this problem relate to the concept of partitions?