CHAPTER 7: THE CENTRAL LIMIT THEOREM

Exercise 1. *Yoonie is a personnel manager in a large corporation. Each month she must review 16 of the employees. From past experience, she has found that the reviews take her approximately four hours each to do with a population standard deviation of 1.2 hours. Let X be the random variable representing the time it takes her to complete one review. Assume X is normally distributed. Let* $\overline{X}$ *be the random variable representing the mean time to complete the 16 reviews. Assume that the 16 reviews represent a random set of reviews.*

What are the mean, the standard deviation, and the sample size?

Solution Mean = 4 hours

Standard deviation = 1.2 hours

Sample size = 16

Exercise 2. *Yoonie is a personnel manager in a large corporation. Each month she must review 16 of the employees. From past experience, she has found that the reviews take her approximately four hours each to do with a population standard deviation of 1.2 hours. Let X be the random variable representing the time it takes her to complete one review. Assume X is normally distributed. Let* $\overline{X}$ *be the random variable representing the mean time to complete the 16 reviews. Assume that the 16 reviews represent a random set of reviews.*

Complete the distributions.

a. X ~ _____(_____,_____)

b. $\overline{X}$ *~ _____(_____,_____)*

Solution

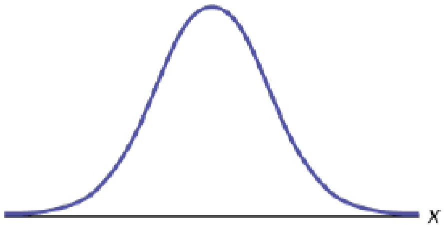
a. $X \sim N(4, 1.2)$

b. $\overline{X} \sim N\left(4, \frac{1.2}{\sqrt{16}}\right)$

Exercise 3. *Yoonie is a personnel manager in a large corporation. Each month she must review 16 of the employees. From past experience, she has found that the reviews take her approximately four hours each to do with a population standard deviation of 1.2 hours. Let X be the random variable representing the time it takes her to complete one review. Assume X is normally distributed. Let* $\overline{X}$ *be the random variable representing the mean time to complete the 16 reviews. Assume that the 16 reviews represent a random set of reviews.*

Find the probability that ***one*** *review will take Yoonie from 3.5 to 4.25 hours. Sketch the graph, labeling and scaling the horizontal axis. Shade the region corresponding to the probability.*

a.



b. P(_____ < x < ______) = ________

Solution

a. Check student's solution.

b. 3.5, 4.25, 0.2441

Exercise 4. *Yoonie is a personnel manager in a large corporation. Each month she must review 16 of the employees. From past experience, she has found that the reviews take her approximately four hours each to do with a population standard deviation of 1.2 hours. Let X be the random variable representing the time it takes her to complete one review. Assume X is normally distributed. Let $\overline{X}$ be the random variable representing the mean time to complete the 16 reviews. Assume that the 16 reviews represent a random set of reviews.*

*Find the probability that the **mean** of a month's reviews will take Yoonie from 3.5 to 4.25 hrs. Sketch the graph, labeling and scaling the horizontal axis. Shade the region corresponding to the probability.*

a. 

b. P(___________) = ________

Solution

a. Check student's solution.

b.3.5, 4.25, 0.7499

Exercise 5. *Yoonie is a personnel manager in a large corporation. Each month she must review 16 of the employees. From past experience, she has found that the reviews take her approximately four hours each to do with a population standard deviation of 1.2 hours. Let X be the random variable representing the time it takes her to complete one review. Assume X is normally distributed. Let $\overline{X}$ be the* *random variable representing the mean time to complete the 16 reviews. Assume that the 16 reviews represent a random set of reviews.*

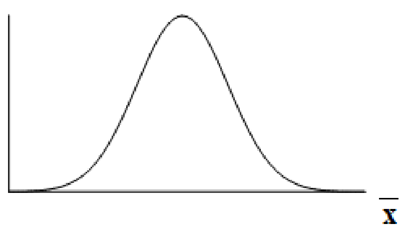
What causes the probabilities in ***Exercise 1.3*** *and* ***Exercise 1.4*** *to be different?*

Solution The fact that the two distributions are different accounts for the different probabilities.

Exercise 6. *Yoonie is a personnel manager in a large corporation. Each month she must review 16 of the employees. From past experience, she has found that the reviews take her approximately four hours each to do with a population standard deviation of 1.2 hours. Let X be the random variable representing the time it takes her to complete one review. Assume X is normally distributed. Let $\overline{X}$ be the random variable representing the mean time to complete the 16 reviews. Assume that the 16 reviews represent a random set of reviews.*

Find the 95th percentile for the mean time to complete one month's reviews. Sketch the graph.

a.



*b. The 95th Percentile =*_______

Solution a. Check student's solution.

b. P$(3.5 < \bar{x} < 4.25) = invNorm\left(0.95, 4, \frac{1.2}{\sqrt{16}}\right) = 4.49$

Exercise 7. *An unknown distribution has a mean of 80 and a standard deviation of 12. A sample size of 95 is drawn randomly from the population.*

Find the probability that the sum of the 95 values is greater than 7,650.

Solution 0.3345

Exercise 8. *An unknown distribution has a mean of 80 and a standard deviation of 12. A sample size of 95 is drawn randomly from the population.*

Find the probability that the sum of the 95 values is less than 7,400.

Solution 0.0436

Exercise 9. *An unknown distribution has a mean of 80 and a standard deviation of 12. A sample size of 95 is drawn randomly from the population.*

Find the sum that is two standard deviations above the mean of the sums.

Solution 7,833.92

Exercise 10. *An unknown distribution has a mean of 80 and a standard deviation of 12. A sample size of 95 is drawn randomly from the population.*

Find the sum that is 1.5 standard deviations below the mean of the sums.

Solution 7,424.56

Exercise 11. *The distribution of results from a cholesterol test has a mean of 180 and a standard deviation of 20. A sample size of 40 is drawn randomly.*

Find the probability that the sum of the 40 values is greater than 7,500.

Solution 0.0089

Exercise 12. *The distribution of results from a cholesterol test has a mean of 180 and a standard deviation of 20. A sample size of 40 is drawn randomly.*

Find the probability that the sum of the 40 values is less than 7,000.

Solution 0.0569

Exercise 13. *The distribution of results from a cholesterol test has a mean of 180 and a standard deviation of 20. A sample size of 40 is drawn randomly.*

Find the sum that is one standard deviation above the mean of the sums.

Solution 7,326.49

Exercise 14. *The distribution of results from a cholesterol test has a mean of 180 and a standard deviation of 20. A sample size of 40 is drawn randomly.*

Find the sum that is 1.5 standard deviations below the mean of the sums.

Solution 7,010.26

Exercise 15. *The distribution of results from a cholesterol test has a mean of 180 and a standard deviation of 20. A sample size of 40 is drawn randomly.*

Find the percentage of sums between 1.5 standard deviations below the mean of the sums and one standard deviation above the mean of the sums

Solution 77.45%

Exercise 16. *A researcher measures the amount of sugar in several cans of the same soda. The mean is 39.01 with a standard deviation of 0.5. The researcher randomly selects a sample of 100.*

Find the probability that the sum of the 100 values is greater than 3,910.

Solution 0.0359

Exercise 17. *A researcher measures the amount of sugar in several cans of the same soda. The mean is 39.01 with a standard deviation of 0.5. The researcher randomly selects a sample of 100.*

Find the probability that the sum of the 100 values is less than 3,900.

Solution 0.4207

Exercise 18. *A researcher measures the amount of sugar in several cans of the same soda. The mean is 39.01 with a standard deviation of 0.5. The researcher randomly selects a sample of 100.*

Find the probability that the sum of the 100 values falls between the numbers you found in exercise 16 and exercise 17.

Solution 0.5433

Exercise 19. *A researcher measures the amount of sugar in several cans of the same soda. The mean is 39.01 with a standard deviation of 0.5. The researcher randomly selects a sample of 100.*

Find the sum with a z-score of –2.5.

Solution 3,888.5

Exercise 20. *A researcher measures the amount of sugar in several cans of the same soda. The mean is 39.01 with a standard deviation of 0.5. The researcher randomly selects a sample of 100.*

Find the sum with a z-score of 0.5.

Solution 3,903.5

Exercise 21. *A researcher measures the amount of sugar in several cans of the same soda. The mean is 39.01 with a standard deviation of 0.5. The researcher randomly selects a sample of 100.*

Find the probability that the sums will fall between the z-scores –2 and 1.

Solution 0.8186

Exercise 22. *An unknown distribution has a mean 12 and a standard deviation of one. A sample size of 25 is taken. Let X = the object of interest.*

What is the mean of ΣX?

Solution 300

Exercise 23. *An unknown distribution has a mean 12 and a standard deviation of one. A sample size of 25 is taken. Let X = the object of interest.*

What is the standard deviation of ΣX?

Solution 5

Exercise 24. *An unknown distribution has a mean 12 and a standard deviation of one. A sample size of 25 is taken. Let X = the object of interest.*

What is P(Σx = 290)?

Solution 0

Exercise 25. *An unknown distribution has a mean 12 and a standard deviation of one. A sample size of 25 is taken. Let X = the object of interest.*

What is $P(\Sigma x > 290)$?

Solution 0.9772

Exercise 26. *True or False: only the sums of normal distributions are also normal distributions.*

Solution False, the sums of any distribution approach a normal distribution as the sample size increases.

Exercise 27. *In order for the sums of a distribution to approach a normal distribution, what must be true?*

Solution The sample size, *n*, gets larger.

Exercise 28. *What three things must you know about a distribution to find the probability of sums?*

Solution the mean of the distribution, the standard deviation of the distribution, and the sample size

Exercise 29. *An unknown distribution has a mean of 25 and a standard deviation of six. Let X = one object from this distribution.*

What is the sample size if the standard deviation of ΣX is 42?

Solution 49

Exercise 30. *An unknown distribution has a mean of 19 and a standard deviation of 20. Let X = the object of interest. What is the sample size if the mean of ΣX is 15,200?*

Solution 800

Exercise 31. *A market researcher analyzes how many electronics devices customers buy in a single purchase. The distribution has a mean of three with a standard deviation of 0.7. She samples 400 customers.*

What is the z-score for Σx = 840?

Solution –26.00

Exercise 32. *A market researcher analyzes how many electronics devices customers buy in a single purchase. The distribution has a mean of three with a standard deviation of 0.7. She samples 400 customers.*

What is the z-score for Σx = 1,186?

Solution –1

Exercise 33. *A market researcher analyzes how many electronics devices customers buy in a single purchase. The distribution has a mean of three with a standard deviation of 0.7. She samples 400 customers.*

What is P(Σx < 1,186)?

Solution 0.1587

Exercise 34. *An unknown distribution has a mean of 100, a standard deviation of 100, and a sample size of 100. Let X = one object of interest.*

What is the mean of *ΣX*?

Solution 10,000

Exercise 35. *An unknown distribution has a mean of 100, a standard deviation of 100, and a sample size of 100. Let X = one object of interest.*

What is the standard deviation of *ΣX*?

Solution 1,000

Exercise 36. *An unknown distribution has a mean of 100, a standard deviation of 100, and a sample size of 100. Let X = one object of interest.*

What is $P(\Sigma x > 9{,}000)$?

Solution 0.8413

Exercise 37. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

a. What is the distribution for the weights of one 25-pound lifting weight? What is the mean and standard deviation?

b. What is the distribution for the mean weight of 100 25-pound lifting weights?

c. Find the probability that the mean actual weight for the 100 weights is less than 24.9.

Solution a. $U(24, 26)$, 25, 0.5774

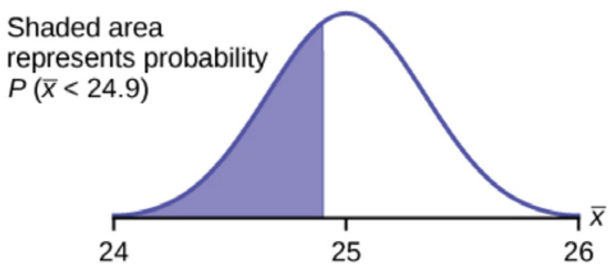
b. $N(25, 0.0577)$

c. 0.0416

Exercise 38. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

*Draw the graph from **Exercise 7.37**.*

Solution



Exercise 39. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

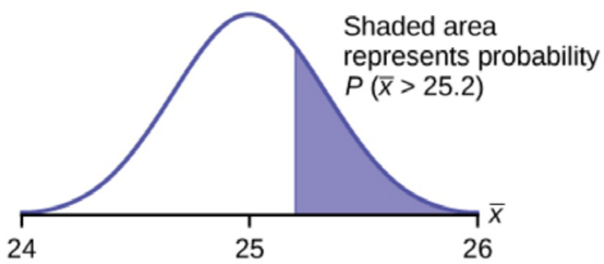
Find the probability that the mean actual weight for the 100 weights is greater than 25.2.

Solution 0.0003

Exercise 40. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

Draw the graph from ***Exercise 7.39****.*

Solution



Exercise 41. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

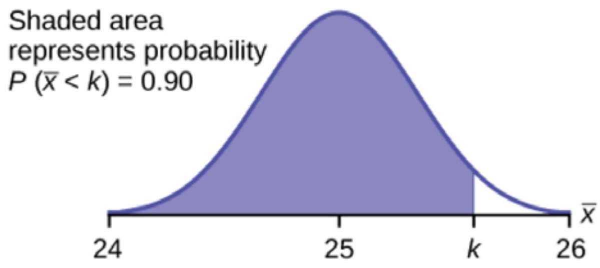
Find the 90th percentile for the mean weight for the 100 weights.

Solution 25.07

Exercise 42. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

*Draw the graph from **Exercise 7.41**.*

Solution



Exercise 43. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

a. What is the distribution for the sum of the weights of 100 25-pound lifting weights?

b. Find P(Σx < 2,450).

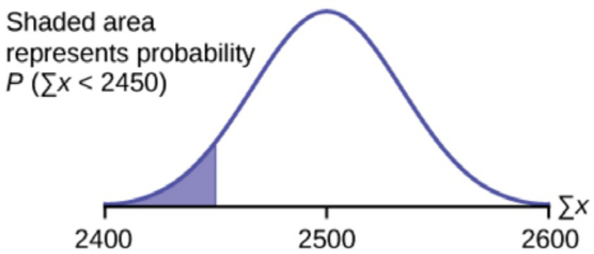
Solution a. *N*(2,500, 5.7735)

b. 0

Exercise 44. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

Draw the graph from ***Exercise 7.43****.*

Solution



Exercise 45. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

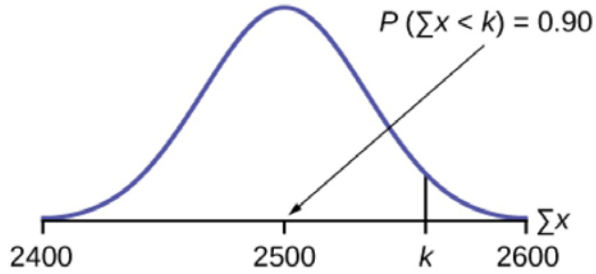
Find the 90th percentile for the total weight of the 100 weights.

Solution 2,507.40

Exercise 46. *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*

Draw the graph from ***Exercise 7.45****.*

Solution



Comment [a1]: CNX – Replace this piece with the revised piece titled “fig-ch06_05_04”

Exercise 47. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

a. What is the standard deviation?

b. What is the parameter m?

Solution

a. 10

b. $\frac{1}{10}$

Exercise 48. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

What is the distribution for the length of time one battery lasts?

Solution

$$Exp\left(\frac{1}{10}\right)$$

Exercise 49. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

What is the distribution for the mean length of time 64 batteries last?

Solution $$N\left(10, \frac{10}{8}\right)$$

Exercise 50. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

What is the distribution for the total length of time 64 batteries last?

Solution $N(640, 80)$

Exercise 51. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

Find the probability that the sample mean is between seven and 11.

Solution 0.7799

Exercise 52. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

Find the 80^{th} percentile for the total length of time 64 batteries last.

Solution 707.3

Exercise 53. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

Find the IQR for the mean amount of time 64 batteries last.

Solution 1.69

Exercise 54. *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*

Find the middle 80% for the total amount of time 64 batteries last.

Solution 205.05

Exercise 55. *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*

Find $P(\Sigma x > 420)$.

Solution 0.0072

Exercise 56. *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*

Find the 90^{th} percentile for the sums.

Solution 410.46

Exercise 57. *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*

Find the 15^{th} percentile for the sums.

Solution 391.54

Exercise 58. *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*

Find the first quartile for the sums.

Solution 394.49

Exercise 59. *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*

Find the third quartile for the sums.

Solution 405.51

Exercise 60. *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*

Find the 80^{th} percentile for the sums.

Solution 406.87

Exercise 61. *Previously, De Anza statistics students estimated that the amount of change daytime statistics students carry is exponentially distributed with a mean of $0.88. Suppose that we randomly pick 25 daytime statistics students.*

a. In words, X = ____________

b. X ~ _____(_____,_____)

c. In words, $\overline{X}$ = ____________

d. $\overline{X}$ ~ ______ (______, ______)

e. Find the probability that an individual had between $0.80 and $1.00. Graph the situation, and shade in the area to be determined.

f. Find the probability that the average of the 25 students was between $0.80 and $1.00. Graph the situation, and shade in the area to be determined.

g. Explain why there is a difference in part e and part f.

Solution a. X = amount of change students carry

b. $X \sim E(0.88, 0.88)$

c. $\overline{X}$ = average amount of change carried by a sample of 25 students.

d. $\overline{X} \sim N(0.88, 0.176)$

e. 0.0819

f. 0.1882

g. The distributions are different. Part a is exponential and part b is normal.

Exercise 62. *Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet. We randomly sample 49 fly balls.*

a. If $\overline{X}$ = average distance in feet for 49 fly balls, then $\overline{X}$ ~ _______(_______,_______)

b. What is the probability that the 49 balls traveled an average of less than 240 feet? Sketch the graph. Scale the horizontal axis for $\overline{X}$. Shade the region corresponding to the probability. Find the probability.

c. Find the 80th percentile of the distribution of the average of 49 fly balls.

Solution a. $N\left(250, \frac{50}{\sqrt{49}}\right)$

b. 0.0808; for graph, check student's solution.

c. 256.01 feet

Exercise 63. *According to the Internal Revenue Service, the average length of time for an individual to complete (keep records for, learn, prepare, copy, assemble, and send) IRS Form 1040 is 10.53 hours (without any attached schedules). The distribution is unknown. Let us assume that the standard deviation is two hours. Suppose we randomly sample 36 taxpayers.*

a. In words, X = ______________

b. In words, $\overline{X}$ = ______________

c. $\overline{X}$ ~_____(_____,_____)

d. Would you be surprised if the 36 taxpayers finished their Form 1040s in an average of more than 12 hours? Explain why or why not in complete sentences.

e. Would you be surprised if one taxpayer finished his or her Form 1040 in more than 12 hours? In a complete sentence, explain why.

Solution

a. length of time for an individual to complete IRS form 1040, in hours.

b. mean length of time for a sample of 36 taxpayers to complete IRS form 1040, in hours.

c. $N(10.53, \frac{1}{3})$

d. Yes. I would be surprised, because the probability is almost 0.

e. No. I would not be totally surprised because the probability is 0.2312

Exercise 64. *Suppose that a category of world-class runners are known to run a marathon (26 miles) in an average of 145 minutes with a standard deviation of 14 minutes. Consider 49 of the races. Let $\overline{X}$ the average of the 49 races.*

a. $\overline{X}$ ~_____(_____,_____)

b. Find the probability that the runner will average between 142 and 146 minutes in these 49 marathons.

c. Find the 80th percentile for the average of these 49 marathons.

d. Find the median of the average running times.

Solution

a. $N\left(145, \frac{14}{\sqrt{49}}\right)$

b. 0.6247

c. 146.68

d. 145 minutes

Exercise 65. *The length of songs in a collector's iTunes album collection is uniformly distributed from two to 3.5 minutes. Suppose we randomly pick five albums from the collection. There are a total of 43 songs on the five albums.*

a. In words, X =_________

b. X ~_____________

c. In words, $\overline{X}$ *=_____________*

d. $\overline{X}$ *~_____(_____,_____)*

e. Find the first quartile for the average song length.

f. The IQR(interquartile range) for the average song length is from _______–_______.

Solution a. the length of a song, in minutes, in the collection

b. *U*(2, 3.5)

c. the average length, in minutes, of the songs from a sample of five albums from collection

d. *N*(2.75, 0.0220)

e. 2.74 minutes

f. 0.03 minutes

Exercise 66. *In 1940 the average size of a U.S. farm was 174 acres. Let's say that the standard deviation was 55 acres. Suppose we randomly survey 38 farmers from 1940.*

a. In words, X =_____________

b. In words, $\overline{X}$ *=_____________*

c. $\overline{X}$ *~_____(_____,_____)*

d. The IQR for $\overline{X}$ *is from _______ acres to _______ acres.*

Solution a. the size of a U.S. farm in 1940

b. the average size of a U.S. farm as estimated from a sample of 38, in acres

c. $N\left(174, \frac{55}{\sqrt{38}}\right)$

d. 168.0, 180.0

Exercise 67. *Determine which of the following are true and which are false. Then, in complete sentences, justify your answers.*

a. When the sample size is large, the mean of $\overline{X}$ is approximately equal to the mean of X.

b. When the sample size is large, $\overline{X}$ is approximately normally distributed.

c. When the sample size is large, the standard deviation of $\overline{X}$ is approximately the same as the standard deviation of X.

Solution a. True. The mean of a sampling distribution of the means is approximately the mean of the data distribution.

b. True. According to the Central Limit Theorem, the larger the sample, the closer the sampling distribution of the means becomes normal.

c. True. The standard deviation of the sampling distribution of the means will decrease making it approximately the same as the standard deviation of *X* as the sample size increases.

Exercise 68. *The percent of fat calories that a person in America consumes each day is normally distributed with a mean of about 36 and a standard deviation of about ten. Suppose that 16 individuals are randomly chosen. Let $\overline{X}$ = average percent of fat calories.*

a. $\overline{X}$ ~ ______(______, ______)

b. For the group of 16, find the probability that the average percent of fat calories consumed is more than five. Graph the situation and shade in the area to be determined.

c. Find the first quartile for the average percent of fat calories.

Solution a. $N\left(36, \frac{10}{\sqrt{16}}\right)$

b. 1

c. 34.31

Exercise 69. *The distribution of income in some Third World countries is considered wedge shaped (many very poor people, very few middle income people, and even fewer wealthy people). Suppose we pick a country with a wedge shaped distribution. Let the average salary be $2,000 per year with a standard deviation of $8,000. We randomly survey 1,000 residents of that country.*

a. In words, X = _____________

b. In words, $\overline{X}$ *= _____________*

c. $\overline{X}$ *~ _____(_____,_____)*

d. How is it possible for the standard deviation to be greater than the average?

e. Why is it more likely that the average of the 1,000 residents will be from $2,000 to $2,100 than from $2,100 to $2,200?

Solution a. *X* = the yearly income of someone in a third world country

b. the average salary from samples of 1,000 residents of a third world country

c. $\bar{X} \sim N\left(2000, \frac{8000}{\sqrt{1000}}\right)$

d. Very wide differences in data values can have averages smaller than standard deviations.

e. The distribution of the sample mean will have higher probabilities closer to the population mean.

$P(2000 < \overline{X} < 2100) = 0.1537$

$P(2100 < \overline{X} < 2200) = 0.1317$

Exercise 70. *Which of the following is NOT TRUE about the distribution for averages?*

a. The mean, median, and mode are equal.

b. The area under the curve is one.

c. The curve never touches the x-axis.

d. The curve is skewed to the right.

Solution d

Exercise 71. *The cost of unleaded gasoline in the Bay Area once followed an unknown distribution with a mean of $4.59 and a standard deviation of $0.10. Sixteen gas stations from the Bay Area are randomly chosen. We are interested in the average cost of gasoline for the 16 gas stations. The distribution to use for the average cost of gasoline for the 16 gas stations is:*

a. $\overline{X} \sim N(4.59, 0.10)$

b. $\overline{X} \sim N\left(4.59, \frac{0.10}{\sqrt{16}}\right)$

c. $\overline{X} \sim N\left(4.59, \frac{16}{0.10}\right)$

d. $\overline{X} \sim N\left(4.59, \frac{16}{0.10}\right)$

Solution b

Exercise 72. *Which of the following is NOT TRUE about the theoretical distribution of sums?*

a. The mean, median and mode are equal.

b. The area under the curve is one.

c. The curve never touches the x-axis.

d. The curve is skewed to the right.

Solution d

Exercise 73. *Suppose that the duration of a particular type of criminal trial is known to have a mean of 21 days and a standard deviation of seven days. We randomly sample nine trials.*

a. In words, ΣX = ______________

b. ΣX ~_____(_____,_____)

c. Find the probability that the total length of the nine trials is at least 225 days.

d. Ninety percent of the total of nine of these types of trials will last at least how long?

Solution a. the total length of time for nine criminal trials

b. *N*(189, 21)

c. 0.0432

d. 162.09; ninety percent of the total nine trials of this type will last 162 days or more.

Exercise 74. *Suppose that the weight of open boxes of cereal in a home with children is uniformly distributed from two to six pounds with a mean of four pounds and standard deviation of 1.1547. We randomly survey 64 homes with children.*

a. In words, X = _____________

b. The distribution is _______.

c. In words, ΣX = _______________

d. ΣX ~_____(_____,_____)

e. Find the probability that the total weight of open boxes is less than 250 pounds.

f. Find the 35^{th} percentile for the total weight of open boxes of cereal.

Solution a. *X* = weight of open cereal boxes

b. The distribution is uniform.

c. ΣX = the sum of the weights of 64 randomly chosen cereal boxes.

d. $N(256, 9.24)$

e. 0.2581

f. 252.44 pounds

Exercise 75. *Salaries for teachers in a particular elementary school district are normally distributed with a mean of $44,000 and a standard deviation of $6,500. We randomly survey ten teachers from that district.*

a. In words, X = _______________

b. X ~ _____(_____,_____)

c. In words, ΣX = ______________

d. ΣX ~ _____(_____,_____)

e. Find the probability that the teachers earn a total of over $400,000.

f. Find the 90th percentile for an individual teacher's salary.

g. Find the 90th percentile for the sum of ten teachers' salary.

h. If we surveyed 70 teachers instead of ten, graphically, how would that change the distribution in part d?

i. If each of the 70 teachers received a $3,000 raise, graphically, how would that change the distribution in part b?

Solution a. X = the salary of one elementary school teacher in the district

b. $X \sim N(44,000, 6,500)$

c. ΣX ~ sum of the salaries of ten elementary school teachers in the sample

d. $\Sigma X \sim N(44000, 20554.80)$

e. 0.9742

f. $52,330.09

g. 466,342.04

h. Sampling 70 teachers instead of ten would cause the distribution to be less spread out. It would be a more symmetrical normal curve.

i. If every teacher received a $3,000 raise, the distribution of *X* would shift to the right by $3,000. In other words, it would have a mean of $47,000.

Exercise 76. *The attention span of a two-year-old is exponentially distributed with a mean of about eight minutes. Suppose we randomly survey 60 two-year-olds.*

a. In words, X = _______

b. X ~_____(_____,_____)

c. In words, $\overline{X}$ = ____________

d. $\overline{X}$ ~_____(_____,_____)

e. Before doing any calculations, which do you think will be higher? Explain why.

i. The probability that an individual attention span is less than ten minutes.

ii. The probability that the average attention span for the 60 children is less than ten minutes?

f. Calculate the probabilities in part e.

g. Explain why the distribution for $\overline{X}$ is not exponential.

Solution a. *X* = the attention span of a two-year-old

b. $X \sim Exp(\frac{1}{8})$

c. $\overline{X}$ = the mean (average) attention span of a two-year-old

d. $\overline{X} \sim N(8\frac{8}{\sqrt{60}})$

e. The standard deviation is smaller, so there is more area under the normal curve. For a sample of 60, as compared to a sample of one (a single individual), the standard deviation will be less and the values will cluster more tightly around the mean, and extreme values will be less common. Therefore, ii is more probable.

f. Exponential: 0.9284

Normal: 0.9736

g. By the central limit theorem, as *n* gets larger, the means tend to follow a normal distribution.

Exercise 77. *The closing stock prices of 35 U.S. semiconductor manufacturers are given as follows. 8.625; 30.25; 27.625; 46.75; 32.875; 18.25; 5; 0.125; 2.9375; 6.875; 28.25; 24.25; 21; 1.5; 30.25; 71; 43.5; 49.25; 2.5625; 31; 16.5; 9.5; 18.5; 18; 9; 10.5; 16.625; 1.25; 18; 12.87; 7; 12.875; 2.875; 60.25; 29.25*

a. In words, X = ______________

b .

i. $\overline{x}$ = _____

ii. s_x = _____

iii. n = _____

c. Construct a histogram of the distribution of the averages. Start at x = –0.0005. Use bar widths of ten.

d. In words, describe the distribution of stock prices.

e. Randomly average five stock prices together. (Use a random number generator.) Continue averaging five pieces together until you have ten averages. List those ten averages.

f. Use the ten averages from part e to calculate the following.

i. $\overline{x}$ = _____

ii. s_x = _____

g. Construct a histogram of the distribution of the averages. Start at x = -0.0005. Use bar widths of ten.

h. Does this histogram look like the graph in part c?

i. In one or two complete sentences, explain why the graphs either look the same or look different?

j. Based upon the theory of the ***central limit theorem,*** *$\overline{X}$ ~ _____(_____,____)* Solution a. X = the closing stock prices for U.S. semiconductor manufacturers

b i. $20.71; ii. $17.31; iii. 35

c. Check student's solution.

d. Exponential distribution, $X \sim Exp(\frac{1}{20.71})$

e. Answers will vary.

f. i. $20.71; ii. $11.14

g. Answers will vary.

h. Answers will vary.

i. Answers will vary.

j. $N(20.71, \frac{17.31}{\sqrt{5}})$

Exercise 78. *Richard's Furniture Company delivers furniture from 10 A.M. to 2 P.M. continuously and uniformly. We are interested in how long (in hours) past the 10 A.M. start time that individuals wait for their delivery.*

X ~_____(_____,_____)

a. U(0,4)

b. U(10,2)

c. Eχp(2)

d. N(2,1)

Solution a

Exercise 79. *Richard's Furniture Company delivers furniture from 10 A.M. to 2 P.M. continuously and uniformly. We are interested in how long (in hours) past the 10 A.M. start time that individuals wait for their delivery.*

The average wait time is:

a. one hour.

b. two hours.

c. two and a half hours.

d. four hours.

Solution b

Exercise 80. *Richard's Furniture Company delivers furniture from 10 A.M. to 2 P.M. continuously and uniformly. We are interested in how long (in hours) past the 10 A.M. start time that individuals wait for their delivery.*

Suppose that it is now past noon on a delivery day. The probability that a person must wait at least one and a half ***more*** *hours is:*

a. $\frac{1}{4}$

b. $\frac{1}{2}$

c. $\frac{3}{4}$

d. $\frac{3}{8}$

Solution a

Exercise 81. *The time to wait for a particular rural bus is distributed uniformly from zero to 75 minutes. One hundred riders are randomly sampled to learn how long they waited.*

The 90^{th} *percentile sample average wait time (in minutes) for a sample of 100 riders is:*

a. 315.0

b. 40.3

c. 38.5

d. 65.2

Solution b

Exercise 82. *The time to wait for a particular rural bus is distributed uniformly from zero to 75 minutes. One hundred riders are randomly sampled to learn how long they waited.*

Would you be surprised, based upon numerical calculations, if the sample average wait time (in minutes) for 100 riders was less than 30 minutes?

a. yes

b. no

c. There is not enough information.

Solution a

Exercise 83. *The cost of unleaded gasoline in the Bay Area once followed an unknown distribution with a mean of $4.59 and a standard deviation of $0.10. Sixteen gas stations from the Bay Area are randomly chosen. We are interested in the average cost of gasoline for the 16 gas stations.*

What's the approximate probability that the average price for 16 gas stations is over $4.69?

a. almost zero

b. 0.1587

c. 0.0943

d. unknown

Solution a

Exercise 84. *The cost of unleaded gasoline in the Bay Area once followed an unknown distribution with a mean of $4.59 and a standard deviation of $0.10. Sixteen gas stations from the Bay Area are randomly chosen. We are interested in the average cost of gasoline for the 16 gas stations.*

Find the probability that the average price for 30 gas stations is less than $4.55.

a. 0.6554

b. 0.3446

c. 0.0142

d. 0.9858

e. 0

Solution c

Exercise 85. *Suppose in a local Kindergarten through 12^{th} grade (K - 12) school district, 53 percent of the population favor a charter school for grades K through five. A simple random sample of 300 is surveyed. Calculate following using the normal approximation to the binomial distribution.*

a. Find the probability that less than 100 favor a charter school for grades K through 5.

b. Find the probability that 170 or more favor a charter school for grades K through 5.

c. Find the probability that no more than 140 favor a charter school for grades K through 5.

d. Find the probability that there are fewer than 130 that favor a charter school for grades K through 5.

e. Find the probability that exactly 150 favor a charter school for grades K through 5.

If you have access to an appropriate calculator or computer software, try calculating these probabilities using the technology.

Solution a. 0

b. 0.1123

c. 0.0162

d. 0.0003

e. 0.0268

Exercise 86. *Four friends, Janice, Barbara, Kathy and Roberta, decided to carpool together to get to school. Each day the driver would be chosen by randomly selecting one of the four names. They carpool to school for 96 days. Use the normal approximation to the binomial to calculate the following probabilities. Round the standard deviation to four decimal places.*

a. Find the probability that Janice is the driver at most 20 days.

b. Find the probability that Roberta is the driver more than 16 days.

c. Find the probability that Barbara drives exactly 24 of those 96 days.

Solution a. 0.2047

b. 0.9615

c. 0.0938

Exercise 87. *$X \sim N(60, 9)$. Suppose that you form random samples of 25 from this distribution. Let $\overline{X}$ be the random variable of averages. Let ΣX be the random variable of sums. For parts c through f, sketch the graph, shade the region, label and scale the horizontal axis for $\overline{X}$, and find the probability.*

a. Sketch the distributions of X and $\overline{X}$ on the same graph.

b. $\overline{X}$ ~ _____(_____,_____)

c. P($\overline{x} < 60$) = _____

d. Find the 30th percentile for the mean.

e. P($56 < \overline{x} < 62$) = _____

f. P($18 < \overline{x} < 58$) = _____

g. Σx ~_____(_____,_____)

h. Find the minimum value for the upper quartile for the sum.

i. $P(1{,}400 < \Sigma x < 1{,}550)$ =_____

Solution a. Check student's solution.

b. $\overline{X} \sim N\left(60, \frac{9}{\sqrt{25}}\right)$

c. 0.5000

d. 59.06

e. 0.8536

f. 0.1333

g. $N(1500,45)$

h.1530.35

i. 0.6877

Exercise 88. *Suppose that the length of research papers is uniformly distributed from ten to 25 pages. We survey a class in which 55 research papers were turned in to a professor. The 55 research papers are considered a random collection of all papers. We are interested in the average length of the research papers.*

a. In words, X =_____________

b. X ~_____(_____,_____)

c. μ_x =_____

d. σ_x =_____

e. In words, $\overline{X}$ =______________

f. $\overline{X}$ ~_____(_____,_____)

g. In words, ΣX =_____________

h. ΣX ~_____(_____,_____)

i. Without doing any calculations, do you think that it's likely that the professor will need to read a total of more than 1,050 pages? Why?

j. Calculate the probability that the professor will need to read a total of more than 1,050 pages.

k. Why is it so unlikely that the average length of the papers will be less than 12 pages?

Solution

a. X = the length of one research paper

b. $U(10, 25)$

c. 17.5

d. $\sqrt{\frac{225}{12}} = 4.3301$

e. $N(17.5, 0.5839)$

f. $N(962.5, 32.11)$

g. 0.0032

h. $N(962.5, 32.11)$

i. Check student's solution.

j. 0.0032

k. The probability is 0.

Exercise 89. *Salaries for teachers in a particular elementary school district are normally distributed with a mean of $44,000 and a standard deviation of $6,500. We randomly survey ten teachers from that district.*

a. Find the 90^{th} percentile for an individual teacher's salary.

b. Find the 90^{th} percentile for the average teacher's salary.

Solution a. $52,330

b. $46,634

Exercise 90. *The average length of a maternity stay in a U.S. hospital is said to be 2.4 days with a standard deviation of 0.9 days. We randomly survey 80 women who recently bore children in a U.S. hospital.*

a. In words, X = _____________

b. In words, $\overline{X}$ = ___________________

c. $\overline{X}$ ~ _____(_____,_____)

d. In words, ΣX = _______________

e. ΣX ~ _____(_____,_____)

f. Is it likely that an individual stayed more than five days in the hospital? Why or why not?

g. Is it likely that the average stay for the 80 women was more than five days? Why or why not?

h. Which is more likely:

i. An individual stayed more than five days.

ii. the average stay of 80 women was more than five days.

i. If we were to sum up the women's stays, is it likely that, collectively they spent more than a year in the hospital? Why or why not?

Solution a. the length of a maternity stay in a U.S. hospital, in days

b. the average length of a maternity stay in a U.S. hospital, in days

c. $\left(5.111, 5.291, 5.201, \frac{0.065}{\sqrt{280}}\right)$

d. *N*(192, 8.05)

e. Not likely, but possible. The mean stay is 2.4 days.

f. No, the probability is 0.

g. Individual

h. No, the probability is 0.

i. Yes, the probability is 1.

Exercise 91. *NeverReady batteries has engineered a newer, longer lasting AAA battery. The company claims this battery has an average life span of 17 hours with a standard deviation of 0.8 hours. Your statistics class questions this claim. As a class, you randomly select 30 batteries and find that the sample mean life span is 16.7 hours. If the process is working properly, what is the probability of getting a random sample of 30 batteries in which the sample mean lifetime is 16.7 hours or less? Is the company's claim reasonable?*

Solution We have μ = 17, σ = 0.8, $\overline{x}$ = 16.7, and n = 30. To calculate the probability, we use normalcdf(lower, upper, μ, $\frac{\sigma}{\sqrt{n}}$) $= \text{normalcdf}\left(E-99, 16.7, 17, \frac{0.8}{\sqrt{30}}\right) = 0.0200$.

• If the process is working properly, then the probability that a sample of 30 batteries would have at most 16.7 lifetime hours is only 2%. Therefore, the class was justified to question the claim.

Exercise 92. *Men have an average weight of 172 pounds with a standard deviation of 29 pounds.*

a. Find the probability that 20 randomly selected men will have a sum weight greater than 3600 lbs.

b. If 20 men have a sum weight greater than 3500 lbs, then their total weight exceeds the safety limits for water taxis. Based on (a), is this a safety concern? Explain.

Solution a. To calculate the probability, we use normalcdf(3600, *E*99, 3440, 129.69) = 0.1087

b. While the probability is not exceptionally large, $P(\overline{X} > 3600)$ = 0.1087, it could be wise to be concerned. After all, we do have a 1 in 10 chance that a random sample of men could have an averages sum weight that exceeds the safety limit.

Exercise 93. *M&M candies large candy bags have a claimed net weight of 396.9 g. The standard deviation for the weight of the individual candies is 0.017 g. The following table is from a stats experiment conducted by a statistics class.*

Red	**Orange**	**Yellow**	**Brown**	**Blue**	**Green**
0.751	0.735	0.883	0.696	0.881	0.925
0.841	0.895	0.769	0.876	0.863	0.914
0.856	0.865	0.859	0.855	0.775	0.881
0.799	0.864	0.784	0.806	0.854	0.865
0.966	0.852	0.824	0.840	0.810	0.865
0.859	0.866	0.858	0.868	0.858	1.015
0.857	0.859	0.848	0.859	0.818	0.876
0.942	0.838	0.851	0.982	0.868	0.809
0.873	0.863			0.803	0.865
0.809	0.888			0.932	0.848
0.890	0.925			0.842	0.940
0.878	0.793			0.832	0.833
0.905	0.977			0.807	0.845
0.850			0.841	0.852	
0.830			0.932	0.778	
0.856			0.833	0.814	
0.842			0.881	0.791	
0.778			0.818	0.810	
0.786			0.864	0.881	

0.853			0.825		
0.864			0.855		
0.873			0.942		
0.880			0.825		
0.882			0.869		
0.931			0.912		
0.887					

The bag contained 465 candies and he listed weights in the table came from randomly selected candies. Count the weights.

a. Find the mean sample weight and the standard deviation of the sample weights of candies in the table.

b. Find the sum of the sample weights in the table and the standard deviation of the sum the of the weights.

c. If 465 M&Ms are randomly selected, find the probability that their weights sum to at least 396.9.

d. Is the Mars Company's M&M labeling accurate?

Solution a. For the sample, we have n = 100, $\overline{x}$ = 0.862, s = 0.05

b. $\Sigma\overline{x}$ = 85.65, Σs = 5.18

c. normalcdf(396.9,*E*99,(465)(0.8565),(0.05) $(\sqrt{465})) \approx 1$

d. Since the probability of a sample of size 465 having at least a mean sum of 396.9 is approximately 1, we can conclude that Mars is correctly labeling their M&M packages.

Exercise 94. *The Screw Right Company claims their* $\frac{3}{4}$ *inch screws are within* ± 0.23 *of the claimed mean diameter of 0.750 inches with a standard deviation of 0.115 inches. The following data were recorded.*

0.757	0.723	0.754	0.737	0.757	0.741	0.722	0.741	0.743	0.742

0.740	0.758	0.724	0.739	0.736	0.735	0.760	0.750	0.759	0.754
0.744	0.758	0.765	0.756	0.738	0.742	0.758	0.757	0.724	0.757
0.744	0.738	0.763	0.756	0.760	0.768	0.761	0.742	0.734	0.754
0.758	0.735	0.740	0.743	0.737	0.737	0.725	0.761	0.758	0.756

The screws were randomly selected from the local home repair store.

a. Find the mean diameter and standard deviation for the sample

b. Find the probability that 50 randomly selected screws will be within the stated tolerance levels. Is the company's diameter claim plausible?

Solution

a. $\overline{x}$ = 0.75 and s 0.01

b. We have normalcdf $\left(0.52,0.98,0.75,\frac{0.115}{\sqrt{50}}\right)$ ≈ 1 and we can conclude that the company's diameter claim is justified.

Exercise 95. *Your company has a contract to perform preventive maintenance on thousands of air-conditioners in a large city. Based on service records from previous years, the time that a technician spends servicing a unit averages one hour with a standard deviation of one hour. In the coming week, your company will service a simple random sample of 70 units in the city. You plan to budget an average of 1.1 hours per technician to complete the work. Will this be enough time?*

Solution

Use normalcdf $(E-99,1.1,1,\frac{1}{\sqrt{70}})$ = 0.7986. This means that there is an 80% chance that the average service time will be less than 1.1 hours. It could be wise to schedule more time since there is an associated 20% chance that the maintenance time will be greater than 1.1 hours.

Exercise 96. *A typical adult has an average IQ score of 105 with a standard deviation of 20. If 20 randomly selected adults are given an IQ test, what is the probability that the sample mean scores will be between 85 and 125 points?*

Solution

The probability that the sample score is between 85 and 125 points is given by normalcdf $(85,125,105,\frac{20}{\sqrt{20}})$ =0.9999. Therefore, it is almost a guarantee that a well selected sample of 20 adults will have an average score between 85 and 125.

Exercise 97. *Certain coins have an average weight of 5.201 grams with a standard deviation of 0.065 g. If a vending machine is designed to accept coins whose weights range from 5.111 g to 5.291 g, what is the expected number of rejected coins when 280 randomly selected coins are inserted into the machine?*

Solution

Since we have normalcdf$(5.111, 5.291, 5.201, \frac{0.065}{\sqrt{280}}) \approx 1$, we can conclude that practically all the coins are within the limits, therefore, there should be no rejected coins out of a well selected sample of size 280.