CHAPTER 8: CONFIDENCE INTERVALS

Exercise 1. The standard deviation of the weights of elephants is known to be approximately 15 pounds. We wish to construct a 95% confidence interval for the mean weight of newborn elephant calves. Fifty newborn elephants are weighed. The sample mean is 244 pounds. The sample standard deviation is 11 pounds.

*Identify the following:*

a. \( \bar{x} = \) _____
b. \( \sigma = \) _____
c. \( n = \) _____

**Solution**

a. 244
b. 15
c. 50

Exercise 2. The standard deviation of the weights of elephants is known to be approximately 15 pounds. We wish to construct a 95% confidence interval for the mean weight of newborn elephant calves. Fifty newborn elephants are weighed. The sample mean is 244 pounds. The sample standard deviation is 11 pounds.

*In words, define the random variables \( X \) and \( \bar{X} \).*

**Solution**

\( X \) is the weight in pounds of a newborn elephant. \( \bar{X} \) is the average of weights of the sample of 50 baby elephants.

Exercise 3. The standard deviation of the weights of elephants is known to be
approximately 15 pounds. We wish to construct a 95% confidence interval for the mean weight of newborn elephant calves. Fifty newborn elephants are weighed. The sample mean is 244 pounds. The sample standard deviation is 11 pounds.

*Which distribution should you use for this problem?*

Solution

\[ N \left( 244, \frac{15}{\sqrt{50}} \right) \]

Exercise 4.

The standard deviation of the weights of elephants is known to be approximately 15 pounds. We wish to construct a 95% confidence interval for the mean weight of newborn elephant calves. Fifty newborn elephants are weighed. The sample mean is 244 pounds. The sample standard deviation is 11 pounds.

*Construct a 95% confidence interval for the population mean weight of newborn elephants. State the confidence interval, sketch the graph, and calculate the error bound.*

Solution

CI: (239.84, 248.16)

\[ EBM = 4.16 \]

Exercise 5.

The standard deviation of the weights of elephants is known to be
approximately 15 pounds. We wish to construct a 95% confidence interval for the mean weight of newborn elephant calves. Fifty newborn elephants are weighed. The sample mean is 244 pounds. The sample standard deviation is 11 pounds.

What will happen to the confidence interval obtained, if 500 newborn elephants are weighed instead of 50? Why?

Solution
As the sample size increases, there will be less variability in the mean and so the interval size decreases.

Exercise 6.
The U.S. Census Bureau conducts a study to determine the time needed to complete the short form. The Bureau surveys 200 people. The sample mean is 8.2 minutes. There is a known standard deviation of 2.2 minutes. The population distribution is assumed to be normal.

Identify the following:

a. $\bar{x} =$ _____
b. $\sigma =$ _____
c. $n =$ _____

Solution
a. $\bar{x} = 8.2$
b. $\sigma = 2.2$
c. $n = 200$

Exercise 7.
The U.S. Census Bureau conducts a study to determine the time needed to complete the short form. The Bureau surveys 200 people. The sample mean is 8.2 minutes. There is a known standard deviation of 2.2 minutes.
The population distribution is assumed to be normal.

In words, define the random variables $X$ and $\bar{X}$.

Solution

$X$ is the time in minutes it takes to complete the U.S. Census short form.

$\bar{X}$ is the mean time it took a sample of 200 people to complete the U.S. Census short form.

Exercise 8.

The U.S. Census Bureau conducts a study to determine the time needed to complete the short form. The Bureau surveys 200 people. The sample mean is 8.2 minutes. There is a known standard deviation of 2.2 minutes. The population distribution is assumed to be normal.

Which distribution should you use for this problem?

Solution

$N(8.2, \frac{2.2}{\sqrt{200}})$

Exercise 9.

The U.S. Census Bureau conducts a study to determine the time needed to complete the short form. The Bureau surveys 200 people. The sample mean is 8.2 minutes. There is a known standard deviation of 2.2 minutes. The population distribution is assumed to be normal.

Construct a 90% confidence interval for the population mean time to complete the forms. State the confidence interval, sketch the graph, and calculate the error bound.

Solution

CI: (7.9441, 8.4559)
Exercise 10.

The U.S. Census Bureau conducts a study to determine the time needed to complete the short form. The Bureau surveys 200 people. The sample mean is 8.2 minutes. There is a known standard deviation of 2.2 minutes. The population distribution is assumed to be normal.

If the Census wants to increase its level of confidence and keep the error bound the same by taking another survey, what changes should it make?

Solution

The Census should survey more people. When we keep the same error bound, the confidence level increases.

Exercise 11.

The U.S. Census Bureau conducts a study to determine the time needed to complete the short form. The Bureau surveys 200 people. The sample mean is 8.2 minutes. There is a known standard deviation of 2.2 minutes. The population distribution is assumed to be normal.

If the Census did another survey, kept the error bound the same, and surveyed only 50 people instead of 200, what would happen to the level of confidence? Why?

Solution

The level of confidence would decrease because decreasing n makes the confidence interval wider, so at the same error bound, the confidence level decreases.
The U.S. Census Bureau conducts a study to determine the time needed to complete the short form. The Bureau surveys 200 people. The sample mean is 8.2 minutes. There is a known standard deviation of 2.2 minutes. The population distribution is assumed to be normal.

Suppose the Census needed to be 98% confident of the population mean length of time. Would the Census have to survey more people? Why or why not?

Solution

The Census would not have to survey more people if they were to increase the error bound and make the confidence interval wider. If they wish to keep the same error bound, then they would have to survey more people.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

Identify the following:

a. $\overline{x} = \underline{\hspace{2cm}}$

b. $\sigma = \underline{\hspace{2cm}}$

c. $n = \underline{\hspace{2cm}}$

Solution

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each
head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

In words, define the random variable $X$.

Solution

$X$ is the weight in pounds of a head of lettuce.

Exercise 15.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

In words, define the random variable $\bar{X}$.

Solution

$\bar{X}$ is the mean weight of a sample of 20 heads of lettuce.

Exercise 16.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

Which distribution should you use for this problem?

Solution

$\mathcal{N}\left(2.2, \frac{0.2}{\sqrt{20}}\right)$

Exercise 17.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each
head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

Construct a 90% confidence interval for the population mean weight of the heads of lettuce. State the confidence interval, sketch the graph, and calculate the error bound.

Solution

\[
CI: (2.1264, 2.2736)
\]

\[
EBM = 0.07
\]

Exercise 18.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

Construct a 95% confidence interval for the population mean weight of the heads of lettuce. State the confidence interval, sketch the graph, and calculate the error bound.

Solution

\[
CI: (2.1123, 2.2877)
\]
Exercise 19

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

In complete sentences, explain why the confidence interval in Exercise 17 is larger than in Exercise 18.

Solution

The interval is greater because the level of confidence increased. If the only change made in the analysis is a change in confidence level, then all we are doing is changing how much area is being calculated for the normal distribution. Therefore, a larger confidence level results in larger areas and larger intervals.

Exercise 20.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

In complete sentences, give an interpretation of what the interval in Exercise 8.18 means.
We can say with 95% confidence that the population mean of the weight of heads of lettuce is between 2.1123 and 2.2877 pounds.

Exercise 21.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

What would happen if 40 heads of lettuce were sampled instead of 20, and the error bound remained the same?

Solution

The confidence level would increase.

Exercise 22.

A sample of 20 heads of lettuce was selected. Assume that the population distribution of head weight is normal. The weight of each head of lettuce was then recorded. The mean weight was 2.2 pounds with a standard deviation of 0.1 pounds. The population standard deviation is known to be 0.2 pounds.

What would happen if 40 heads of lettuce were sampled instead of 20, and the confidence level remained the same?

Solution

The confidence interval would become narrower.

Exercise 23. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean
age for Winter Foothill College students. Let \( X = \) the age of a Winter Foothill College student

\[ \bar{x} = 30.4 \]

Solution 30.4

Exercise 24. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let \( X = \) the age of a Winter Foothill College student

\[ n = 25 \]

Solution 25

Exercise 25. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let \( X = \) the age of a Winter Foothill College student

\[ \sigma = 15 \]

Solution \( \sigma \)

Exercise 26. The mean age for all Foothill College students for a recent Fall term
was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X = \text{the age of a Winter Foothill College student}$

In words, define the random variable $\bar{X}$.

**Solution**

The mean age of 25 randomly selected Winter Foothill students

**Exercise 27.**

The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X = \text{the age of a Winter Foothill College student}$

What is $\bar{x}$ estimating?

**Solution**

$\mu$

**Exercise 28.**

The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X = \text{the age of a Winter Foothill College student}$

Is $\sigma_x$ known?
Exercise 29. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let \( X \) = the age of a Winter Foothill College student

As a result of your answer to Exercise 8.26, state the exact distribution to use when calculating the confidence interval.

Solution

normal

Exercise 30. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let \( X \) = the age of a Winter Foothill College student

Construct a 95% Confidence Interval for the true mean age of Winter Foothill College students.

How much area is in both tails (combined)? \( \alpha = \) ____

Solution

0.05

Exercise 31. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent
Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let \( X = \) the age of a Winter Foothill College student.

Construct a 95% Confidence Interval for the true mean age of Winter Foothill College students.

How much area is in each tail? \( \frac{\alpha}{2} = \) _______

**Solution**

0.025

**Exercise 32.**

The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let \( X = \) the age of a Winter Foothill College student.

Construct a 95% Confidence Interval for the true mean age of Winter Foothill College students.

Identify the following specifications:

a. lower limit
b. upper limit
c. error bound

**Solution**

a. 24.52
b. 36.28
c. 5.88
Exercise 33. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X = \text{the age of a Winter Foothill College student}$

Construct a 95% Confidence Interval for the true mean age of Winter Foothill College students.

The 95% confidence interval is:__________________

Solution (24.52 ,36.28)

Exercise 34. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X = \text{the age of a Winter Foothill College student}$

Construct a 95% Confidence Interval for the true mean age of Winter Foothill College students.

Fill in the blanks on the graph with the areas, upper and lower limits of the confidence interval, and the sample mean.
Exercise 35. The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X =$ the age of a Winter Foothill College student.
Construct a 95% Confidence Interval for the true mean age of Winter Foothill College students.

In one complete sentence, explain what the interval means.

Solution

We are 95% confident that the true mean age for Winger Foothill College students is between 24.52 and 36.28.

Exercise 36.

The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X$ = the age of a Winter Foothill College student

Using the same mean, standard deviation and level of confidence, suppose that $n$ were 69 instead of 25. Would the error bound become larger or smaller? How do you know?

Solution

The error bound for the mean would become smaller because as sample sizes increase, if all other factors remain unchanged, variability decreases. And with less variability you need a smaller interval to capture the true population mean.

Exercise 37.

The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. Let $X$ = the age of a Winter Foothill College student
Using the same mean, standard deviation, and sample size, how would the error bound change if the confidence level were reduced to 90%? Why?

Solution
The error bound for the mean would decrease because as the CL decreases, you need less area under the normal curve (which translates into a smaller interval) to capture the true population mean.

Exercise 38.
A hospital is trying to cut down on emergency room wait times. It is interested in the amount of time patients must wait before being called back to be examined. An investigation committee randomly surveyed 70 patients. The sample mean was 1.5 hours with a sample standard deviation of 0.5 hours.

Identify the following:

a. $\bar{x} = ____$

b. $s_x = ____$

c. $n = ____$

d. $n - 1 = ____$

Solution
a. $\bar{x} = 1.5$

b. $s_x = 0.5$

c. $n = 70$

d. $n - 1 = 69$

Exercise 39.
A hospital is trying to cut down on emergency room wait times. It is interested in the amount of time patients must wait before being called back to be examined. An investigation committee randomly surveyed
70 patients. The sample mean was 1.5 hours with a sample standard deviation of 0.5 hours.

Define the random variables $X$ and $\overline{X}$, in words.

Solution $X$ is the number of hours a patient waits in the emergency room before being called back to be examined. $\overline{X}$ is the mean wait time of 70 patients in the emergency room.

Exercise 40. A hospital is trying to cut down on emergency room wait times. It is interested in the amount of time patients must wait before being called back to be examined. An investigation committee randomly surveyed 70 patients. The sample mean was 1.5 hours with a sample standard deviation of 0.5 hours.

Which distribution should you use for this problem?

Solution Student’s $t$: $t_{69}$

Exercise 41 A hospital is trying to cut down on emergency room wait times. It is interested in the amount of time patients must wait before being called back to be examined. An investigation committee randomly surveyed 70 patients. The sample mean was 1.5 hours with a sample standard deviation of 0.5 hours.

Construct a 95% confidence interval for the population mean time spent waiting. State the confidence interval, sketch the graph, and calculate the error bound.

Solution CI: (1.3808, 1.6192)
Exercise 42. A hospital is trying to cut down on emergency room wait times. It is interested in the amount of time patients must wait before being called back to be examined. An investigation committee randomly surveyed 70 patients. The sample mean was 1.5 hours with a sample standard deviation of 0.5 hours.

Explain in complete sentences what the confidence interval means.

Solution We are 95% confident that the population mean waiting time is between 1.38 hours and 1.62 hours for patients in the emergency room before getting called back to be examined.

Exercise 43. One hundred eight Americans were surveyed to determine the number of hours they spend watching television each month. It was revealed that they watched an average of 151 hours each month with a standard deviation of 32 hours. Assume that the underlying population distribution is normal.

Identify the following:

a. \( \bar{x} = \) _____

b. \( s_x = \) _____

c. \( n = \) _____

d. \( n - 1 = \) _____

\( EBM = 0.12 \)
Exercise 44. One hundred eight Americans were surveyed to determine the number of hours they spend watching television each month. It was revealed that they watched an average of 151 hours each month with a standard deviation of 32 hours. Assume that the underlying population distribution is normal.

Define the random variable $X$ in words.

Solution

$X$ is the number of hours per month an American watches television.

Exercise 45. One hundred eight Americans were surveyed to determine the number of hours they spend watching television each month. It was revealed that they watched an average of 151 hours each month with a standard deviation of 32 hours. Assume that the underlying population distribution is normal.

Define the random variable $\bar{X}$ in words.

Solution

$\bar{X}$ is the mean number of hours spent watching television per month from a sample of 108 Americans.

Exercise 46. One hundred eight Americans were surveyed to determine the number of hours they spend watching television each month. It was revealed that they watched an average of 151 hours each month with a
standard deviation of 32 hours. Assume that the underlying population distribution is normal.

Which distribution should you use for this problem?

Solution

Student’s t: \( t_{107} \)

Exercise 47. One hundred eight Americans were surveyed to determine the number of hours they spend watching television each month. It was revealed that they watched an average of 151 hours each month with a standard deviation of 32 hours. Assume that the underlying population distribution is normal.

Construct a 99% confidence interval for the population mean hours spent watching television per month. (a) State the confidence interval, (b) sketch the graph, and (c) calculate the error bound.

Solution

CI: (142.92, 159.08)

\[ EBM = 8.08 \]

Exercise 48. One hundred eight Americans were surveyed to determine the number of hours they spend watching television each month. It was revealed that they watched an average of 151 hours each month with a standard deviation of 32 hours. Assume that the underlying population distribution is normal.
Why would the error bound change if the confidence level were lowered to 95%?

Solution

The error bound would decrease because less variation would have to be accounted for in the confidence interval, so the confidence interval would narrow.

Exercise 49.

The data in Table 8.9 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let \( X \) = the number of colors on a national flag.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1.10

Calculate the following

a. \( \bar{x} = \) ______

b. \( s_x = \) ______

c. \( n = \) ______

Solution

a. 3.26

b. 1.02

c. 39
Exercise 50. The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let \( X \) = the number of colors on a national flag.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1.10

Define the random variable \( \bar{X} \) in words.

Solution the mean number of colors of 39 flags

Exercise 51. The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let \( X \) = the number of colors on a national flag.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 1.10

<table>
<thead>
<tr>
<th>x</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

*Table 1.10*

**What is estimating?**

Solution: 

$$\mu$$

**Exercise 52.** The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $$X$$ be the number of colors on a national flag.

Is $$\sigma_x$$ known?
Solution

Exercise 53. The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $X = \text{the number of colors on a national flag}$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1.10

As a result of your answer to Exercise 52, state the exact distribution to use when calculating the confidence interval.

Solution $t_{38}$

Exercise 54. The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $X = \text{the number of colors on a national flag}$.
Exercise 55. \textit{The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let } X \text{ = the number of colors on a national flag.}
Table 8.10

Construct a 95% confidence interval for the true mean number of colons on national flags.

How much area is in each tail?

Solution

0.025

Exercise 56. The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $X$ = the number of colors on a national flag.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 8.10

Construct a 95% confidence interval for the true mean number of
colons on national flags.

Calculate the following:

a. lower limit

b. upper limit

c. error bound

Solution

a. 2.93

b. 3.59

c. 0.33

Exercise 57.
The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let \( X \) = the number of colors on a national flag.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 8.10
Construct a 95% confidence interval for the true mean number of colons on national flags.

The 95% confidence interval is _____.

Solution

(2.93, 3.59)

Exercise 58.

The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let \( X \) = the number of colors on a national flag.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

**Table 8.10**

Construct a 95% confidence interval for the true mean number of colons on national flags.

Fill in the blanks on the graph with the areas, the upper and lower limits of the Confidence Interval and the sample mean.
Solution

Exercise 59. The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $X =$ the number of colors on a national flag.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>
Table 8.10

Construct a 95% confidence interval for the true mean number of colons on national flags.

In one complete sentence, explain what the interval means.

Solution

We are 95% confident that the true mean number of colors for national flags is between 2.93 colors and 3.59 colors.

Exercise 60.

The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $X =$ the number of colors on a national flag.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 8.10

Construct a 95% confidence interval for the true mean number of colors on national flags.
Using the same $\bar{x}$, $s_x$, and level of confidence, suppose that $n$ were 69 instead of 39. Would the error bound become larger or smaller? How do you know?

Solution

The error bound would become $EBM = 0.245$. This error bound decreases because as sample sizes increase, variability decreases and we need less interval length to capture the true mean.

Exercise 61.

The data in Table 8.10 are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $X =$ the number of colors on a national flag.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 8.10

Construct a 95% confidence interval for the true mean number of colors on national flags.

Using the same $\bar{x}, s_x, n = 39$, how would the error bound change if the confidence level were reduced to 90%. Why?
The confidence interval would become (2.98, 3.54). With a decrease in confidence level, we do not need as large an interval to capture the true population mean. In other words, lower confidence levels demand less certainty.

Exercise 62. *Marketing companies are interested in knowing the population percent of women who make the majority of household purchasing decisions. When designing a study to determine this population proportion, what is the minimum number you would need to survey to be 90% confident that the population proportion is estimated to within 0.05?*

Solution 271

Exercise 63. *Marketing companies are interested in knowing the population percent of women who make the majority of household purchasing decisions. If it were later determined that it was important to be more than 90% confident and a new survey were commissioned, how would it affect the minimum number you need to survey? Why?*

Solution It would decrease, because the z-score would decrease, which reducing the numerator and lowering the number.

Exercise 64. *Suppose the marketing company did do a survey. They randomly surveyed 200 households and found that in 120 of them, the woman made the majority of the purchasing decisions. We are interested in the population proportion of households where women make the majority of the purchasing decisions.*
Identify the following:

a. $x = \underline{\hspace{2cm}}$

b. $n = \underline{\hspace{2cm}}$

c. $p' = \underline{\hspace{2cm}}$

Solution

a. $x = 120$

b. $n = 200$

c. $p' = 0.6$

Exercise 65. Suppose the marketing company did do a survey. They randomly surveyed 200 households and found that in 120 of them, the woman made the majority of the purchasing decisions. We are interested in the population proportion of households where women make the majority of the purchasing decisions.

Define the random variables $X$ and $P'$ in words.

Solution

$X$ is the number of “successes” where the woman makes the majority of the purchasing decisions for the household. $P'$ is the percentage of households sampled where the woman makes the majority of the purchasing decisions for the household.

Exercise 66. Suppose the marketing company did do a survey. They randomly surveyed 200 households and found that in 120 of them, the woman made the majority of the purchasing decisions. We are interested in the population proportion of households where women make the majority
of the purchasing decisions.

Which distribution should you use for this problem?

Solution

\[ N \left( 0.6, \sqrt{\frac{(0.6)(0.4)}{200}} \right) \]

Exercise 67. Suppose the marketing company did do a survey. They randomly surveyed 200 households and found that in 120 of them, the woman made the majority of the purchasing decisions. We are interested in the population proportion of households where women make the majority of the purchasing decisions.

Construct a 95% confidence interval for the population proportion of households where the women make the majority of the purchasing decisions. State the confidence interval, sketch the graph, and calculate the error bound.

Solution

CI: (0.5321, 0.6679)

\[ EBM: 0.0679 \]

Exercise 68. Suppose the marketing company did do a survey. They randomly surveyed 200 households and found that in 120 of them, the woman made the majority of the purchasing decisions. We are interested in the
population proportion of households where women make the majority of the purchasing decisions.

List two difficulties the company might have in obtaining random results, if this survey were done by email.

Solution

Emails are self-selected, so only households that choose to respond can be recorded. This rules out households that do not have internet access or do not have email addresses.

Exercise 69.

Of 1,050 randomly selected adults, 360 identified themselves as manual laborers, 280 identified themselves as non-manual wage earners, 250 identified themselves as mid-level managers, and 160 identified themselves as executives. In the survey, 82% of manual laborers preferred trucks, 62% of non-manual wage earners preferred trucks, 54% of mid-level managers preferred trucks, and 26% of executives preferred trucks.

We are interested in finding the 95% confidence interval for the percent of executives who prefer trucks. Define random variables X and P' in words.

Solution

X is the number of “successes” where an executive prefers a truck. P' is the percentage of executives sampled who prefer a truck.

Exercise 70.

Of 1,050 randomly selected adults, 360 identified themselves as manual laborers, 280 identified themselves as non-manual wage earners, 250 identified themselves as mid-level managers, and 160 identified themselves as executives. In the survey, 82% of manual laborers preferred trucks, 62% of non-manual wage earners preferred...
trucks, 54% of mid-level managers preferred trucks, and 26% of executives preferred trucks.

*Which distribution should you use for this problem?*

**Solution**

\[
N \left( 0.26, \sqrt{\frac{(0.26)(0.74)}{160}} \right)
\]

**Exercise 71.**

Of 1,050 randomly selected adults, 360 identified themselves as manual laborers, 280 identified themselves as non-manual wage earners, 250 identified themselves as mid-level managers, and 160 identified themselves as executives. In the survey, 82% of manual laborers preferred trucks, 62% of non-manual wage earners preferred trucks, 54% of mid-level managers preferred trucks, and 26% of executives preferred trucks.

Construct a 95% confidence interval. State the confidence interval, sketch the graph, and calculate the error bound.

**Solution**

CI: (0.19432, 0.33068)

\[EBM: 0.0707\]

**Exercise 72.**

Of 1,050 randomly selected adults, 360 identified themselves as manual laborers, 280 identified themselves as non-manual wage earners, 250
identified themselves as mid-level managers, and 160 identified themselves as executives. In the survey, 82% of manual laborers preferred trucks, 62% of non-manual wage earners preferred trucks, 54% of mid-level managers preferred trucks, and 26% of executives preferred trucks.

Suppose we want to lower the sampling error. What is one way to accomplish that?

Solution

We increase the sample size.

Exercise 73.

Of 1,050 randomly selected adults, 360 identified themselves as manual laborers, 280 identified themselves as non-manual wage earners, 250 identified themselves as mid-level managers, and 160 identified themselves as executives. In the survey, 82% of manual laborers preferred trucks, 62% of non-manual wage earners preferred trucks, 54% of mid-level managers preferred trucks, and 26% of executives preferred trucks.

The sampling error given in the survey is ±2%. Explain what the ±2% means.

Solution

The sampling error means that the true mean can be 2% above or below the sample mean.

Exercise 74.

A poll of 1,200 voters asked what the most significant issue was in the upcoming election. Sixty-five percent answered the economy. We are interested in the population proportion of voters who feel the economy is the most important.

Define the random variable X in words.

Solution

X is the number of voters who said the economy is the most important
issue in the upcoming election.

Exercise 75. A poll of 1,200 voters asked what the most significant issue was in the upcoming election. Sixty-five percent answered the economy. We are interested in the population proportion of voters who feel the economy is the most important.

Define the random variable $P'$, in words.

Solution $P'$ is the proportion of voters sampled who said the economy is the most important issue in the upcoming election.

Exercise 76 A poll of 1,200 voters asked what the most significant issue was in the upcoming election. Sixty-five percent answered the economy. We are interested in the population proportion of voters who feel the economy is the most important.

Which distribution should you use for this problem?

Solution $N\left(0.65, \sqrt{\frac{(0.65)(0.35)}{1,200}}\right)$

Exercise 77 A poll of 1,200 voters asked what the most significant issue was in the upcoming election. Sixty-five percent answered the economy. We are interested in the population proportion of voters who feel the economy is the most important.

Construct a 90% confidence interval and state the confidence interval and
the error bound.

Solution

CI: (0.62735, 0.67265)  

EBM: 0.02265

Exercise 78  

A poll of 1,200 voters asked what the most significant issue was in the upcoming election. Sixty-five percent answered the economy. We are interested in the population proportion of voters who feel the economy is the most important.

What would happen to the confidence interval if the level of confidence were 95%?

Solution

The confidence interval would get wider because it would have to account for more variation.

Exercise 79  

The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

What is being counted?

Solution

The number of girls, ages 8 to 12, in the 5 P.M. Monday night beginning ice-skating class
Exercise 80

*The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.*

*In words, define the random variable X.*

Solution

The number of girls, ages 8 to 12, in the beginning ice-skating class

Exercise 81

*The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.*

*Calculate the following:*

- a. \( x = \) _____
- b. \( n = \) _____
- c. \( p' = \) _____

Solution

a. \( x = 64 \)

b. \( n = 80 \)
c. $p' = 0.8$

**Exercise 82**  
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

State the estimated distribution of $X$. $X \sim \underline{\text{______}}$

**Solution**  
$B(80, 0.80)$

**Exercise 83**  
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

Define a new random variable $P'$. What is $p'$ estimating?

**Solution**  
$p$

**Exercise 84**  
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls
and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

In words, define the random variable $P'$.

**Solution**
The proportion of girls, ages 8 to 12, in the beginning ice skating class.

**Exercise 85**
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

State the estimated distribution of $P'$. Construct a 92% Confidence Interval for the true proportion of girls in the ages 8 - 12 beginning ice-skating classes at the Ice Chalet.

**Solution**
\[ P' \sim N \left( 0.8, \sqrt{\frac{(0.8)(0.2)}{80}} \right) \approx (0.72171, 0.87829) \]

**Exercise 86**
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume
that the 80 children in the selected class are a random sample of the population.

How much area is in both tails (combined)?

Solution

1 − 0.92 = 0.08

Exercise 87

The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

How much area is in each tail?

Solution

0.04

Exercise 88

The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

Calculate the following:

a. lower limit = _____
Exercise 89
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the 80 children in the selected class are a random sample of the population.

The 92% confidence interval is ______.

Solution
(0.72, 0.88)

Exercise 90
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the children in the selected class are a random sample of the population.

Fill in the blanks on the graph with the areas, upper and lower limits of the
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the children in the selected class are a random sample of the population.

In one complete sentence, explain what the interval means.
Solution

With 92% confidence, we estimate the proportion of girls, ages 8 to 12, in a beginning ice-skating class at the Ice Chalet to be between 72% and 88%.

Exercise 92

*The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the children in the selected class are a random sample of the population.*

*Using the same $p'$ and level of confidence, suppose that $n$ were increased to 100. Would the error bound become larger or smaller? How do you know?*

Solution

The error bound would become smaller. Since $n$ is in the denominator in the formula for the error, increasing the sample size increases the value in the denominator, which decreases the error.

Exercise 93

*The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the children in the selected class are a random sample of the population.*

*Using the same $p'$ and $n = 80$, how would the error bound change if the*
confidence level were increased to 98%? Why?

Solution
The error bound would increase. Assuming all other variables are kept constant, as the confidence level increases, the area under the curve corresponding to the confidence level becomes larger, which creates a wider interval and thus a larger error.

Exercise 94
The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 to 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 to 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the children in the selected class are a random sample of the population.

If you decreased the allowable error bound, why would the minimum sample size increase (keeping the same level of confidence)?

Solution
There are only two variables that can be adjusted in the calculation of the error bound: the critical value, \( z\frac{\alpha}{2} \), which increases as the confidence level increases, and the sample size, \( n \). Thus, since \( n \) is in the denominator, it must be increased in order to decrease the error bound to the maximum value allowed.

Exercise 95.
Among various ethnic groups, the standard deviation of heights is known to be approximately three inches. We wish to construct a 95% confidence interval for the mean height of male Swedes. Forty-eight male Swedes are surveyed. The sample mean is 71 inches. The sample standard deviation is 2.8 inches.
a.
   i. \( \bar{x} = \) 
   ii. \( \sigma = \) 
   iii. \( n = \) 

b. In words, define the random variables, \( X \) and \( \bar{X} \), in words.
c. Which distribution should you use for this problem? Explain your choice.
d. Construct a 95% confidence interval for the population mean height of male Swedes.
   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.
e. What will happen to the level of confidence obtained if 1,000 male Swedes are surveyed instead of 48? Why?

Solution
a.
   i. 71
   ii. 3
   iii. 48
b. \( X \) is the height of a Swiss male, and \( \bar{X} \) is the mean height from a sample of 48 Swiss males.
c. Normal. We know the standard deviation for the population, and the sample size is greater than 30.
d.
   i. CI: (70.151, 71.49)
   ii.
iii. EBM = 0.849

e. The confidence interval will decrease in size, because the sample size increased. Recall, when all factors remain unchanged, an increase in sample size decreases variability. Thus, we do not need as large an interval to capture the true population mean.

Exercise 96.  
Announcements for 84 upcoming engineering conferences were randomly picked from a stack of IEEE Spectrum magazines. The mean length of the conferences was 3.94 days, with a standard deviation of 1.28 days. Assume the underlying population is normal.

a. In words, define the random variables $X$ and $\bar{X}$.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 95% confidence interval for the population mean length of engineering conferences.

i. State the confidence interval.
ii. Sketch the graph.

iii. Calculate the error bound.

Solution

a. $X$ is length of an engineering conference. $\bar{X}$ is the mean length from a sample of 84 engineering conferences.

b. Normal distribution $\mathcal{N}(3.94, \frac{1.28}{\sqrt{84}})$. We know the standard deviation for the population and the sample size is greater than 30.

c.

i. $(3.67, 4.21)$

ii. Check student’s solution.

iii. EBM = 0.2737

Exercise 97.

Suppose that an accounting firm does a study to determine the time needed to complete one person’s tax forms. It randomly surveys 100 people. The sample mean is 23.6 hours. There is a known standard deviation of 7.0 hours. The population distribution is assumed to be normal.

a.

i. $\bar{x} =$ _____

ii. $\sigma =$ _____

iii. $n =$ _____

b. In words, define the random variables $X$ and $\bar{X}$. 
c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 90% confidence interval for the population mean time to complete the tax forms.
   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

e. If the firm wished to increase its level of confidence and keep the error bound the same by taking another survey, what changes should it make?

f. If the firm did another survey, kept the error bound the same, and only surveyed 49 people, what would happen to the level of confidence? Why?

g. Suppose that the firm decided that it needed to be at least 96% confident of the population mean length of time to within one hour. How would the number of people the firm surveys change? Why?

Solution

a.
   i. \( \bar{x} = 23.6 \)
   ii. \( \sigma = 7 \)
   iii. \( n = 100 \)

b. \( X \) is the time needed to complete an individual tax form. \( \bar{X} \) is the mean time to complete tax forms from a sample of 100 customers.

c. \( \mathcal{N}(23.6, \frac{7}{\sqrt{100}}) \) because we know sigma.

d.
   i. \((22.228, 24.972)\)
e. It will need to change the sample size. The firm needs to determine what the confidence level should be, then apply the error bound formula to determine the necessary sample size.

f. The confidence level would decrease as a result of the same size error bound (and thus, the same size interval) and a smaller sample size.

g. According to the error bound formula, the firm needs to survey 206 people. Since we increase the confidence level, we need to increase either our error bound or the sample size.

Exercise 98.

A sample of 16 small bags of the same brand of candies was selected. Assume that the population distribution of bag weights is normal. The weight of each bag was then recorded. The mean weight was two ounces with a standard deviation of 0.12 ounces. The population standard deviation is known to be 0.1 ounce.

a.

i. \( \bar{x} = \) ________
ii. \( \sigma = \) ________

iii. \( s_x = \) ________

b. In words, define the random variable \( X \).

c. In words, define the random variable \( \bar{X} \).

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 90% confidence interval for the population mean weight of the candies.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound

f. Construct a 98% confidence interval for the population mean weight of the candies.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound

g. In complete sentences, explain why the confidence interval in part f is larger than the confidence interval in part e.

h. In complete sentences, give an interpretation of what the interval in part f means.

Solution

a.

i. 2

ii. 0.1

iii. 0.12

b. The weight of a single bag of candy

c. The mean weight from a sample of 16 bags of candies
d. \( N\left(2, \frac{0.1}{\sqrt{16}}\right) \)

e.

i. CI: (1.96, 2.04)

ii.

iii. EBM: 0.04

f.

i. CI: (1.94, 2.06)

ii.

iii. EBM: 0.06

g. A confidence interval is determined by a bounded area under a normal curve. If all the constraints remain unchanged except the level of confidence, then the area that is associated with the increased confidence level also increases.

h. We are 98% confident that the interval from 1.94 to 2.06 contains
the true population mean weight for the candies.

Exercise 99. A camp director is interested in the mean number of letters each child sends during his or her camp session. The population standard deviation is known to be 2.5. A survey of 20 campers is taken. The mean from the sample is 7.9 with a sample standard deviation of 2.8.

a.

i. \( \bar{x} = \) ________

ii. \( \sigma = \) ________

iii. \( n = \) ________

b. Define the random variables \( X \) and \( \bar{X} \) in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 90% confidence interval for the population mean number of letters campers send home.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound

e. What will happen to the error bound and confidence interval if 500 campers are surveyed? Why?

Solution

a.

i. 7.9

ii. 2.5

iii. 20

b. \( X \) is the number of letters a single camper will send home. \( \bar{X} \) is the mean number of letters sent home from a sample of 20 campers.
c. $N \left( 7.9, \frac{2.5}{\sqrt{20}} \right)$

d.

i. CI: (6.98, 8.82)

ii.

iii. $EBM: 0.92$

e. The error bound and confidence interval will decrease.

Exercise 100.

What is meant by the term “90% confident” when constructing a confidence interval for a mean?

a. If we took repeated samples, approximately 90% of the samples would produce the same confidence interval.

b. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the sample mean.

c. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the true value of the population mean.

d. If we took repeated samples, the sample mean would equal the population mean in approximately 90% of the samples.
Exercise 101. The Federal Election Commission collects information about campaign contributions and disbursements for candidates and political committees each election cycle. During the 2012 campaign season, there were 1,619 candidates for the House of Representatives across the United States who received contributions from individuals. Table 1.11 shows the total receipts from individuals for a random selection of 40 House candidates rounded to the nearest $100. The standard deviation for this data to the nearest hundred is $σ = 909,200.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$3,600</td>
<td>$1,243,900</td>
<td>$10,900</td>
<td>$385,200</td>
<td>$581,500</td>
</tr>
<tr>
<td>$7,400</td>
<td>$2,900</td>
<td>$400</td>
<td>$3,714,500</td>
<td>$632,500</td>
</tr>
<tr>
<td>$391,000</td>
<td>$467,400</td>
<td>$56,800</td>
<td>$5,800</td>
<td>$405,200</td>
</tr>
<tr>
<td>$733,200</td>
<td>$8,000</td>
<td>$468,700</td>
<td>$75,200</td>
<td>$41,000</td>
</tr>
<tr>
<td>$13,300</td>
<td>$9,500</td>
<td>$953,800</td>
<td>$1,113,500</td>
<td>$1,109,300</td>
</tr>
<tr>
<td>$353,900</td>
<td>$986,100</td>
<td>$88,600</td>
<td>$378,200</td>
<td>$13,200</td>
</tr>
<tr>
<td>$3,800</td>
<td>$745,100</td>
<td>$5,800</td>
<td>$3,072,100</td>
<td>$1,626,700</td>
</tr>
<tr>
<td>$512,900</td>
<td>$2,309,200</td>
<td>$6,600</td>
<td>$202,400</td>
<td>$15,800</td>
</tr>
</tbody>
</table>

Table 1.11

a. Find the point estimate for the population mean.

b. Using 95% confidence, calculate the error bound.

c. Create a 95% confidence interval for the mean total individual contributions.

d. Interpret the confidence interval in the context of the problem.
Solution

a. $\bar{x} = 568,873$

b. $CL = 0.95 \quad \alpha = 1 - 0.95 = 0.05 \quad z_\alpha = 1.96$

$$EBM = z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{909200}{\sqrt{40}} = 281,764$$

c. $\bar{x} - EBM = 568,873 - 281,764 = 287,109$

$\bar{x} + EBM = 568,873 + 281,764 = 850,637$

Alternate solution:

Using the TI-83, 83+, 84, 84+ Calculators

1. Press STAT and arrow over to TESTS.
2. Arrow down to 7:ZInterval.
3. Press ENTER.
4. Arrow to Stats and press ENTER.
5. Arrow down and enter the following values:
   
   $\sigma : 909,200$
   
   $\bar{x} : 568,873$
   
   $n : 40$
   
   $CL: 0.95$

6. Arrow down to Calculate and press ENTER.
7. The confidence interval is ($287,114, 850,632$).
8. Notice the small difference between the two solutions—these differences are simply due to rounding error in the hand calculations.

d. We estimate with 95% confidence that the mean amount of contributions received from all individuals by House candidates is between $287,109$ and $850,637$.

Exercise 102.

The American Community Survey (ACS), part of the United States Census Bureau, conducts a yearly census similar to the one taken every ten years, but with a smaller percentage of participants. The most
recent survey estimates with 90% confidence that the mean household income in the U.S. falls between $69,720 and $69,922. Find the point estimate for mean U.S. household income and the error bound for mean U.S. household income.

Solution

One solution:

Point estimate $= \frac{(\text{lower bound} + \text{upper bound})}{2} = \frac{(69,720 + 69,922)}{2} = $69,821

Error bound = upper bound - point estimate = 69,922 - 69,821 = $101

Alternate solution:

Error bound $= \frac{(\text{upper bound} - \text{lower bound})}{2} = \frac{(69,922 - 69,720)}{2} = $101

Point estimate = upper bound - error bound = 69,922 - 101 = $69,821

Exercise 103. The average height of young adult males has a normal distribution with standard deviation of 2.5 inches. You want to estimate the mean height of students at your college or university to within one inch with 93% confidence. How many male students must you measure?

Solution

Use the formula for EBM, solved for $n$:

$n = \frac{z^2\sigma^2}{EBM^2}$ From the statement of the problem, you know that $\sigma = 2.5$, and you need $EBM = 1$. $z = z_{0.035} = 1.812$ (This is the value of $z$ for which the area under the density curve to the right of $z$ is 0.035.)

$n = \frac{z^2\sigma^2}{EBM^2} = \frac{1.812^2 \cdot 2.5^2}{1^2} \approx 20.52$ You need to measure at least 21 male students to achieve your goal.

Exercise 104. In six packages of “The Flintstones® Real Fruit Snacks” there were five Bam-Bam snack pieces. The total number of snack pieces in the six bags
was 68. We wish to calculate a 96% confidence interval for the population proportion of Bam-Bam snack pieces.

a. Define the Random Variables X and P' in words.
b. Which distribution should you use for this problem? Explain your choice.
c. Calculate p’.
d. Construct a 96% confidence interval for the population proportion of Bam-Bam snack pieces per bag.
   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.
e. Do you think that six packages of fruit snacks yield enough data to give accurate results? Why or why not?

Solution

a. P is the proportion of Bam-Bam snacks in a bag of snacks. P’ is the proportion of Bam-Bam pieces in a sample of 68 snack pieces.

b. We need to use a one-sample-z-interval which uses a Normal distribution.

c. p’ = 5/68 = 0.074

d.
   i. (0.009, 0.139)

ii.
Exercise 105.

A random survey of enrollment at 35 community colleges across the United States yielded the following figures: 6,414; 1,550; 2,109; 9,350; 21,828; 4,300; 5,944; 5,722; 2,825; 2,044; 5481; 5,200; 5,853; 2,750; 10,012; 6,357; 27,000; 9,414; 7,681; 3,200; 17,500; 9,200; 7,380; 18,314; 6,557; 13,713; 17,768; 7,493; 2,771; 2,861; 1,263; 7,285; 28,165; 5,080; 11,622. Assume the underlying population is normal.

a.
   i. $\bar{x} =$
   ii. $s_x =$
   iii. $n =$
   iv. $n - 1 =$

b. Define the Random Variables $X$ and $\bar{x}$ in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population mean enrollment at community colleges in the United States.
   i. State the confidence interval.
ii. Sketch the graph.

iii. Calculate the error bound.

e. What will happen to the error bound and confidence interval if 500 community colleges were surveyed? Why?

Solution

a. 

i. 8,629

ii. 6944

iii. 35

iv. 34

b. \( X \) is the enrollment for a community college. \( \bar{X} \) is the mean enrollment from a sample of 35 community colleges.

c. We will use a Student’s \( t \)-distribution, \( t_{34} \), because we do not know the population standard deviation.

d.

i. CI: (6244 ,11014)

ii.
e. Both will decrease in size. Remember, any time the sample size increases by a large amount, variability decreases. When variability decreases, we need smaller CIs with smaller error bounds to capture the true population parameter.

Exercise 106. 

Suppose that a committee is studying whether or not there is waste of time in our judicial system. It is interested in the mean amount of time individuals waste at the courthouse waiting to be called for jury duty. The committee randomly surveyed 81 people who recently served as jurors. The sample mean wait time was eight hours with a sample standard deviation of four hours.

a. 

i. \( \bar{x} = \)________

ii. \( s_x = \)________

iii. \( n = \)________

iv. \( n - 1 = \)________

b. Define the random variables \( X \) and \( \bar{X} \) in words.
c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population mean time wasted.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

e. Explain in a complete sentence what the confidence interval means.

Solution

a.

i. 8

ii. 4

iii. 81

iv. 80

b. \( X \) is the amount of time an individual waits at the courthouse to be called for service. \( \bar{X} \) is the mean wait time for a sample size of 81.

c. We need to use the Student’s t-distribution, because we do not know the population standard deviation.

d.

i. CI: (7.12, 8.88)

ii.
iii. \( EBM: 0.89 \)

e. We are 95% confident that the interval between 7.12 minutes and 8.88 minutes captures the true mean wait time at a courthouse.

**Exercise 107.**

A pharmaceutical company makes tranquilizers. It is assumed that the distribution for the length of time they last is approximately normal. Researchers in a hospital used the drug on a random sample of nine patients. The effective period of the tranquilizer for each patient (in hours) was as follows: 2.7; 2.8; 3.0; 2.3; 2.3; 2.2; 2.8; 2.1; and 2.4.

a.

i. \( \bar{x} = \) 

ii. \( s_x = \) 

iii. \( n = \) 

iv. \( n - 1 = \) 

b. Define the random variable \( X \) in words.

c. Define the random variable \( \bar{X} \) in words.

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 95% confidence interval for the population mean length of time.

i. State the confidence interval.
ii. Sketch the graph.

iii. Calculate the error bound.

f. What does it mean to be “95% confident” in this problem?

Solution

a.

i. $\bar{x} = 2.51$

ii. $s_x = 0.318$

iii. $n = 9$

iv. $n - 1 = 8$

b. the effective length of time for a tranquilizer

c. the mean effective length of time of tranquilizers from a sample of nine patients

d. We need to use a Student’s-t distribution, because we do not know the population standard deviation.

e.

i. CI: (2.27, 2.76)

ii. Check student’s solution.

iii. $EBM: 0.25$

f. If we were to sample many groups of nine patients, 95% of the samples would contain the true population mean length of time.

Exercise 108.

Suppose that 14 children, who were learning to ride two-wheel bikes, were surveyed to determine how long they had to use training wheels. It was revealed that they used them an average of six months with a sample standard deviation of three months. Assume that the underlying
The population distribution is normal.

a.
   i. \( \bar{x} = \) ______
   ii. \( s_x = \) ______
   iii. \( n = \) ______
   iv. \( n - 1 = \) ______

b. Define the random variable \( X \) in words.

c. Define the random variable \( \bar{X} \) in words.

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 99% confidence interval for the population mean length of time for using training wheels.
   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

f. Why would the error bound change if the confidence level were lowered to 90%?

Solution

a.
   i. \( \bar{x} = 6 \)
   ii. \( s_x = 3 \)
   iii. \( n = 14 \)
   iv. \( n - 1 = 13 \)

b. the amount of time a single child uses training wheels
c. the mean length of time for training wheels usage from a sample of 14 children
d. We will use a Student’s-t distribution since we do not know the population standard deviation.
e.
Exercise 109.

The Federal Election Commission (FEC) collects information about campaign contributions and disbursements for candidates and political committees each election cycle. A political action committee (PAC) is a committee formed to raise money for candidates and campaigns. A Leadership PAC is a PAC formed by a federal politician (senator or representative) to raise money to help other candidates’ campaigns.

The FEC has reported financial information for 556 Leadership PACs that were operating during the 2011-2012 election cycle. The following table shows the total receipts during this cycle for a random selection of 20 Leadership PACs.

<table>
<thead>
<tr>
<th>Receipts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$46,500.00</td>
</tr>
<tr>
<td>$0</td>
</tr>
<tr>
<td>$40,966.50</td>
</tr>
<tr>
<td>$105,887.20</td>
</tr>
<tr>
<td>$5,175.00</td>
</tr>
<tr>
<td>$29,050.00</td>
</tr>
<tr>
<td>$19,500.00</td>
</tr>
<tr>
<td>$181,557.20</td>
</tr>
<tr>
<td>$31,500.00</td>
</tr>
<tr>
<td>$149,970.80</td>
</tr>
<tr>
<td>$2,555,363.20</td>
</tr>
<tr>
<td>$12,025.00</td>
</tr>
<tr>
<td>$409,000.00</td>
</tr>
<tr>
<td>$60,521.70</td>
</tr>
<tr>
<td>$18,000.00</td>
</tr>
<tr>
<td>$61,810.20</td>
</tr>
<tr>
<td>$76,530.80</td>
</tr>
<tr>
<td>$119,459.20</td>
</tr>
<tr>
<td>$0</td>
</tr>
<tr>
<td>$63,520.00</td>
</tr>
<tr>
<td>$6,500.00</td>
</tr>
<tr>
<td>$502,578.0</td>
</tr>
<tr>
<td>$0</td>
</tr>
<tr>
<td>$705,061.10</td>
</tr>
<tr>
<td>$708,258.90</td>
</tr>
<tr>
<td>$135,810.00</td>
</tr>
<tr>
<td>$2,000.00</td>
</tr>
<tr>
<td>$2,000.00</td>
</tr>
<tr>
<td>$0</td>
</tr>
<tr>
<td>$80</td>
</tr>
<tr>
<td>$1,287,933.80</td>
</tr>
<tr>
<td>$219,148.30</td>
</tr>
</tbody>
</table>

\[ \bar{x} = \$251,854.23 \]

\[ s = \$521,130.41 \]

Use this sample data to construct a 96% confidence interval for the mean
amount of money raised by all Leadership PACs during the 2011–2012 election cycle. Use the Student’s-t distribution.

\[ \bar{x} = \$251,854.23 \quad s = \$521,130.41 \]

Note that we are not given the population standard deviation, only the standard deviation of the sample. There are 30 measures in the sample, so \( n = 30 \), and \( df = 30 - 1 = 29 \)

\[ CL = 0.96, \quad \alpha = 1 - CL = 1 - 0.96 = 0.04 \]

\[ \frac{\alpha}{2} = 0.02 \]

\[ t_{\frac{\alpha}{2}} = t_{0.02} = 2.150 \]

\[ EBM = t_{\frac{\alpha}{2}} \cdot \left( \frac{s}{\sqrt{n}} \right) = 2.150 \cdot \frac{521,130.41}{\sqrt{30}} \sim \$204,561.66 \]

\[ \bar{x} - EBM = \$251,854.23 - \$204,561.66 = \$47,292.57 \]

\[ \bar{x} + EBM = \$251,854.23 + \$204,561.66 = \$456,415.89 \]

We estimate with 96% confidence that the mean amount of money raised by all Leadership PACs during the 2011–2012 election cycle lies between \$47,292.57 and \$456,415.89.

Alternate Solution

**Using the TI-83, 83+, 84, 84+ Calculators**

Enter the data as a list.

Press STAT and arrow over to TESTS.

Arrow down to 8:TInterval.

Press ENTER.

Arrow to Data and press ENTER.

Arrow down and enter the name of the list where the data is stored.

Enter Freq: 1

Enter C-Level: 0.96

Arrow down to Calculate and press Enter.

The 96% confidence interval is (\$47,262, \$456,447).

The difference between solutions arises from rounding differences.
Forbes magazine published data on the best small firms in 2012. These were firms that had been publicly traded for at least a year, have a stock price of at least $5 per share, and have reported annual revenue between $5 million and $1 billion. The Table 1.12 shows the ages of the corporate CEOs for a random sample of these firms.

<table>
<thead>
<tr>
<th>48</th>
<th>58</th>
<th>51</th>
<th>61</th>
<th>56</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>74</td>
<td>63</td>
<td>53</td>
<td>50</td>
</tr>
<tr>
<td>59</td>
<td>60</td>
<td>60</td>
<td>57</td>
<td>46</td>
</tr>
<tr>
<td>55</td>
<td>63</td>
<td>57</td>
<td>47</td>
<td>55</td>
</tr>
<tr>
<td>57</td>
<td>43</td>
<td>61</td>
<td>62</td>
<td>49</td>
</tr>
<tr>
<td>67</td>
<td>67</td>
<td>55</td>
<td>55</td>
<td>49</td>
</tr>
</tbody>
</table>

Use this sample data to construct a 90% confidence interval for the mean age of CEOs for these top small firms. Use the Student’s-t distribution.

Solution

Begin by calculating the point estimate: \( \bar{x} = 56.567 \)

Next, calculate the standard deviation of the sample: \( s = 6.907 \)

\( n = 30, \) and \( df = 30 - 1 = 29 \)

\( CL = 0.90, \) so \( \alpha = 1 - CL = 1 - 0.90 = 0.10 \)

\( \alpha / 2 = 0.05 \)

\( t_{\alpha / 2} = t_{0.05} = 1.699 \)

\[ EBM = t_{\alpha / 2} \cdot \left( \frac{s}{\sqrt{n}} \right) = 1.699 \cdot \frac{6.907}{\sqrt{30}} = 2.143 \]

\( \bar{x} - EBM = 56.567 - 2.146 = 54.421 \)

\( \bar{x} + EBM = 56.567 + 2.146 = 58.713 \)

We estimate with 90% confidence that the mean age of all CEOs of the best small firms lies between 54.421 years and 58.713 years.

Alternate Solution

Using the TI-83, 83+, 84, 84+ Calculators

Enter the data as a list.

Press STAT and arrow over to TESTS.

Arrow down to 8:TInterval.
Press ENTER.
Arrow to Data and press ENTER.
Arrow down and enter the name of the list where the data is stored.
Enter Freq: 1
Enter C-Level: .9
Arrow down to Calculate and press Enter.
The 90% confidence interval is (51.751, 57.249).

Exercise 111.

Unoccupied seats on flights cause airlines to lose revenue. Suppose a large airline wants to estimate its mean number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected and the number of unoccupied seats is noted for each of the sampled flights. The sample mean is 11.6 seats and the sample standard deviation is 4.1 seats.

a.
   i. \( \bar{x} = \) _____
   ii. \( s_x = \) _____
   iii. \( n = \) _____
   iv. \( n - 1 = \) _____

b. Define the random variables \( X \) and \( \bar{X} \) in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 92% confidence interval for the population mean number of unoccupied seats per flight.
   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

Solution
   i. \( \bar{x} = 11.6 \)
\( \text{ii. } s_x = 4.1 \)
\( \text{iii. } n = 225 \)
\( \text{iv. } n - 1 = 224 \)

b. \( X \) is the number of unoccupied seats on a single flight. \( \bar{X} \) is the mean number of unoccupied seats from a sample of 225 flights.

c. We will use a Student’s-\( t \) distribution, because we do not know the population standard deviation.

d. 
\( \text{i. CI: (11.12 , 12.08) } \)
\( \text{ii. Check student’s solution. } \)
\( \text{iii. EBM: 0.48 } \)

Exercise 112. In a recent sample of 84 used car sales costs, the sample mean was $6,425 with a standard deviation of $3,156. Assume the underlying distribution is approximately normal.

a. Which distribution should you use for this problem? Explain your choice.

b. Define the random variable \( \bar{X} \) in words

c. Construct a 95% confidence interval for the population mean cost of a used car.
   \( \text{i. State the confidence interval. } \)
   \( \text{ii. Sketch the graph. } \)
   \( \text{iii. Calculate the error bound. } \)

d. Explain what a “95% confidence interval” means for this study.
Solution

a. We will use a Student’s-t distribution, because we do not know the population standard deviation. The distribution is $t_{83}$

b. mean cost of 84 used cars

c.

i. CI: (5,740.10, 7,109.90)

ii. Check student’s solution.

iii. $EBM = 684.90$

d. If we were to sample $n = 84$ many times, we would expect to see 95% of the sample contain the true population mean selling price of a used car.

Exercise 113.

*Six different national brands of chocolate chip cookies were randomly selected at the supermarket. The grams of fat per serving are as follows: 8; 8; 10; 7; 9; 9. Assume the underlying distribution is approximately normal.*

a. Construct a 90% confidence interval for the population mean grams of fat per serving of chocolate chip cookies sold in supermarkets.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

b. If you wanted a smaller error bound while keeping the same level of confidence, what should have been changed in the study before it was done?

c. Go to the store and record the grams of fat per serving of six brands of chocolate chip cookies.
d. Calculate the mean.

e. Is the mean within the interval you calculated in part a? Did you expect it to be? Why or why not?

Solution

a.

i. CI: (7.64, 9.36)

ii.

iii. EBM: 1.13

b. The sample should have been increased.

c. Answers will vary.

d. Answers will vary.

e. Answers will vary.

Exercise 114.

A survey of the mean number of cents off that coupons give was conducted by randomly surveying one coupon per page from the coupon sections of a recent San Jose Mercury News. The following data
were collected: 20¢; 75¢; 50¢; 65¢; 30¢; 55¢; 40¢; 40¢; 30¢; 55¢; $1.50; 40¢; 65¢; 40¢. Assume the underlying distribution is approximately normal.

a.

i. $\bar{x} =$

ii. $s_x =$

iii. $n =$

iv. $n - 1 =$

b. Define the Random Variables $X$ and $\bar{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population mean worth of coupons.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

e. If many random samples were taken of size 14, what percent of the confidence intervals constructed should contain the population mean worth of coupons? Explain why.

Solution

a.

i. $\bar{x} = 0.539$

ii. $s_x = 0.316$

iii. $n = 14$

iv. $n - 1 = 13$

b. $X$ is the discount amount of a single coupon. $\bar{X}$ is the mean discount amount from a sample of 14 coupons.

c. We need to use a Student’s-$t$ distribution, because we do not know the
population standard deviation.

d.
  i. CI: (0.357, 0.722)
  ii.

iii. \( EBM: 0.183 \)

e. We would expect 95% of the samples to contain the true population mean, because this is how level of confidence is defined.

Exercise 115. A quality control specialist for a restaurant chain takes a random sample of size 12 to check the amount of soda served in the 16 oz. serving size. The sample mean is 13.30 with a sample standard deviation of 1.55. Assume the underlying population is normally distributed.

Find the 95% Confidence Interval for the true population mean for the amount of soda served.

a. (12.42, 14.18)

b. (12.32, 14.29)

c. (12.50, 14.10)
Exercise 116. A quality control specialist for a restaurant chain takes a random sample of size 12 to check the amount of soda served in the 16 oz. serving size. The sample mean is 13.30 with a sample standard deviation of 1.55. Assume the underlying population is normally distributed. What is the error bound?

a. 0.87  
b. 1.98  
c. 0.99  
d. 1.74

Solution c

Exercise 117. Insurance companies are interested in knowing the population percent of drivers who always buckle up before riding in a car.

a. When designing a study to determine this population proportion, what is the minimum number you would need to survey to be 95% confident that the population proportion is estimated to within 0.03?

b. If it were later determined that it was important to be more than 95% confident and a new survey was commissioned, how would that affect the minimum number you would need to survey? Why?

Solution a. 1,068

b. The sample size would need to be increased since the critical value
Suppose that the insurance companies did do a survey. They randomly surveyed 400 drivers and found that 320 claimed they always buckle up. We are interested in the population proportion of drivers who claim they always buckle up.

a.  

i = __________  

ii. n = __________  

iii. p' = __________  

b. Define the random variables X and P', in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population proportion who claim they always buckle up.

i. State the confidence interval.  

ii. Sketch the graph.  

iii. Calculate the error bound.

e. If this survey were done by telephone, list three difficulties the companies might have in obtaining random results.

Solution  

a.  

i. 320
ii. 400

iii. 0.80

b. $X =$ the number of drivers who claim they always buckle up; $P' =$ the proportion of drivers in a sample who claim they always buckle up

c. $N \left( 0.80, \sqrt{\frac{(0.80)(0.20)}{400}} \right)$

d.

i. CI: (0.76, 0.84)

ii. Check student’s solution.

iii. EB: 0.04

e. Not everyone has a phone or a listed telephone number; many people refuse to take surveys over the phone or may not be honest in their answers; many people screen their calls and won’t answer the phone if they don’t recognize the caller’s number.

Exercise 119. According to a recent survey of 1,200 people, 61% feel that the president is doing an acceptable job. We are interested in the population proportion of people who feel the president is doing an acceptable job.

a. Define the random variables $X$ and $P'$ in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 90% confidence interval for the population proportion of
people who feel the president is doing an acceptable job.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

Solution

a. \( X \) = the number of people who feel that the president is doing an acceptable job;

\( P' \) = the proportion of people in a sample who feel that the president is doing an acceptable job.

b. \( N \left( 0.61, \sqrt{\frac{(0.61)(0.39)}{1200}} \right) \)

c. i. CI: (0.59, 0.63)

ii. Check student’s solution

iii. EB: 0.02

Exercise 120. An article regarding interracial dating and marriage recently appeared in the Washington Post. Of the 1,709 randomly selected adults, 315 identified themselves as Latinos, 323 identified themselves as blacks, 254 identified themselves as Asians, and 779 identified themselves as whites. In this survey, 86% of blacks said that they would welcome a white person into their families. Among Asians, 77% would welcome a white person into their families, 71% would welcome a Latino, and 66% would welcome a black person.

a. We are interested in finding the 95% confidence interval for the
percent of all black adults who would welcome a white person into their families. Define the random variables $X$ and $P'$, in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 95% confidence interval

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

Solution

a. $X = \text{the number of black adults who say their families would welcome a white person into their families;}$ $P' = \text{the proportion of black adults who say their families would welcome a white person into their families}$

b. $N\left(0.86, \sqrt{\frac{(0.86)(0.14)}{323}}\right)$

c.

i. CI: (0.823, 0.898)

ii. Check student’s solution.

iii. $EBM: 0.038$

Exercise 121. An article regarding interracial dating and marriage recently appeared in the Washington Post. Of the 1,709 randomly selected adults, 315 identified themselves as Latinos, 323 identified themselves as blacks, 254 identified themselves as Asians, and 779 identified themselves as
whites. In this survey, 86% of blacks said that they would welcome a
white person into their families. Among Asians, 77% would welcome a
white person into their families, 71% would welcome a Latino, and 66%
would welcome a black person.

a. Construct three 95% confidence intervals.

i. percent of all Asians who would welcome a white person into their
families

ii. percent of all Asians who would welcome a Latino into their families

iii. percent of all Asians who would welcome a black person into their
families

b. Even though the three point estimates are different, do any of the
confidence intervals overlap? Which?

c. For any intervals that do overlap, in words, what does this imply
about the significance of the differences in the true proportions?

d. For any intervals that do not overlap, in words, what does this imply
about the significance of the differences in the true proportions?

Solution

a.

i. (0.72, 0.82)

ii. (0.65, 0.76)

iii. (0.60, 0.72)

b. Yes, the intervals (0.72, 0.82) and (0.65, 0.76) overlap, and the
intervals (0.65, 0.76) and (0.60, 0.72) overlap.
c. We can say that there does not appear to be a significant difference between the proportion of Asian adults who say that their families would welcome a white person into their families and the proportion of Asian adults who say that their families would welcome a Latino person into their families.

d. We can say that there is a significant difference between the proportion of Asian adults who say that their families would welcome a white person into their families and the proportion of Asian adults who say that their families would welcome a black person into their families.

Exercise 122.

Stanford University conducted a study of whether running is healthy for men and women over age 50. During the first eight years of the study, 1.5% of the 451 members of the 50-Plus Fitness Association died. We are interested in the proportion of people over 50 who ran and died in the same eight-year period.

a. Define the random variables \( X \) and \( P' \), in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 97% confidence interval for the population proportion of people over 50 who ran and died in the same eight–year period.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

d. Explain what a “97% confidence interval” means for this study.
Solution

a. \( X \) = the number of people over age 50 who ran and died in the same eight-year period as the study; \( P' \) = the proportion of people over age 50 in a sample who ran and died in the same eight-year period as the study.

b. Since we are estimating a proportion, given \( p' = 0.015 \) and \( n = 451 \), the distribution we should use is \( N \left( 0.015, \sqrt{\frac{(0.015)(0.985)}{451}} \right) \).

c.

i. CI: (0.003, 0.028)

ii. Check student’s solution.

iii. \( EBM: 0.0125 \)

d. We can be 97% confident that the proportion of people over age 50 in the population who ran and died in the same eight-year period as the study is between 0.3% and 2.8%.

Exercise 123. A telephone poll of 1,000 adult Americans was reported in an issue of Time Magazine. One of the questions asked was “What is the main problem facing the country?” Twenty percent answered “crime.” We are interested in the population proportion of adult Americans who feel that crime is the main problem.

a. Define the random variables \( X \) and \( P' \) in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 95% confidence interval for the population proportion of
adult Americans who feel that crime is the main problem.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

d. Suppose we want to lower the sampling error. What is one way to accomplish that?

e. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is ± 3%. In one to three complete sentences, explain what the ± 3% represents.

Solution

a. \( X \) = the number of adult Americans who feel that crime is the main problem; \( P' \) = the proportion of adult Americans who feel that crime is the main problem

b. Since we are estimating a proportion, given \( p' = 0.2 \) and \( n = 1,000 \), the distribution we should use is \( N \left( 0.2, \sqrt{\frac{0.2(0.8)}{1000}} \right) \).

c.

i. CI: (0.18, 0.22)

ii. Check student’s solution.

iii. \( EBM \): 0.02

d. One way to lower the sampling error is to increase the sample size.

e. The stated “± 3%” represents the maximum error bound. This means that those doing the study are reporting a maximum error of 3%. Thus,
they estimate the percentage of adult Americans who feel that crime is the main problem to be between 18% and 22%.

Exercise 124.

A telephone poll of 1,000 adult Americans was reported in an issue of Time Magazine. One of the questions asked was “What is the main problem facing the country?” 20% answered “crime.” Another question in the poll was “[How much are] you worried about the quality of education in our schools?” Sixty-three percent responded “a lot.” We are interested in the population proportion of adult Americans who are worried a lot about the quality of education in our schools.

a. Define the random variables \( X \) and \( \hat{P} \), in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 95% confidence interval for the population proportion of adult Americans who are worried a lot about the quality of education in our schools.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the error bound.

d. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is ± 3%. In one to three complete sentences, explain what the ± 3% represents.

Solution

\( a. X = \) the number of adult Americans who are worried a lot about the quality of education in our schools; \( \hat{P} = \) the proportion of adult
Americans in a sample who are worried a lot about the quality of education in our schools

b. Since we are estimating a proportion, given \( p' = 0.63 \) and \( n = 1000 \), so the distribution we should use is \( N \left( 0.63, \frac{0.63 \times 0.37}{1000} \right) \).

c.

i. (0.60, 0.66)

ii. Check student’s solution.

iii. EBM = 0.03

Exercise 125. According to a Field Poll, 79% of California adults (actual results are 400 out of 506 surveyed) feel that “education and our schools” is one of the top issues facing California. We wish to construct a 90% confidence interval for the true proportion of California adults who feel that education and the schools is one of the top issues facing California.

A point estimate for the true population proportion is:

a. 0.90

b. 1.27

c. 0.79

d. 400

Solution c

Exercise 126. According to a Field Poll, 79% of California adults (actual results are
400 out of 506 surveyed) feel that “education and our schools” is one of the top issues facing California. We wish to construct a 90% confidence interval for the true proportion of California adults who feel that education and the schools is one of the top issues facing California.

A 90% confidence interval for the population proportion is:

a. (0.761, 0.820)

b. (0.125, 0.188)

c. (0.755, 0.826)

d. (0.130, 0.183)

Solution a

Exercise 127. According to a Field Poll, 79% of California adults (actual results are 400 out of 506 surveyed) feel that “education and our schools” is one of the top issues facing California. We wish to construct a 90% confidence interval for the true proportion of California adults who feel that education and the schools is one of the top issues facing California.

The error bound is approximately

a. 1.581

b. 0.791

c. 0.059

d. 0.030

Solution d
Exercise 128. Five hundred eleven (511) homes in a certain southern California community are randomly surveyed to determine if they meet minimal earthquake preparedness recommendations. One hundred seventy-three (173) of the homes surveyed met the minimum recommendations for earthquake preparedness and 338 did not.

Find the confidence interval at the 90% Confidence Level for the true population proportion of southern California community homes meeting at least the minimum recommendations for earthquake preparedness.

a. (0.2975, 0.3796)

b. (0.6270, 0.6959)

c. (0.3041, 0.3730)

d. (0.6204, 0.7025)

Solution c

Exercise 129 Five hundred eleven (511) homes in a certain southern California community are randomly surveyed to determine if they meet minimal earthquake preparedness recommendations. One hundred seventy-three (173) of the homes surveyed met the minimum recommendations for earthquake preparedness and 338 did not.

The point estimate for the population proportion of homes that do not meet the minimum recommendations for earthquake preparedness is:

a. 0.6614
Exercise 130.

On May 23, 2013, Gallup reported that of 1,005 people surveyed, 76% of U. S. workers believe that they will continue working past retirement age. The confidence level for this study was reported at 95% with a ±3% margin of error.

a. Determine the estimated proportion from the sample.
b. Determine the sample size.
c. Identify CL and \( \alpha \).
d. Calculate the error bound based on the information provided.
e. Compare the error bound in part d to the margin of error reported by Gallup. Explain any differences between the values.
f. Create a confidence interval for the results of this study.
g. A reporter is covering the release of this study for a local news station. How should she explain the confidence interval to her audience?

Solution

a. \( p' = 0.76 \), the proportion of participants in the study who believe that they will work beyond retirement age

b. \( n = 1,005 \)

c. \( CL = 0.95 \), so \( \alpha = 1 - 0.95 = 0.05 \)

d. \( EPB = \left( \frac{z}{2} \right) \sqrt{\frac{p'q'}{n}} = (1.96) \sqrt{\frac{0.76(0.24)}{1005}} \approx 0.026 \) \( \left( \frac{\alpha}{2} = 0.025 \right) \)

The area to the right of this \( z \) is 0.025, so the area
to the left is $1 - 0.025 = 0.975$.)

e. Gallup reports a 3% margin of error, or an error bound of 0.03. The error bound calculated from the details provided is 0.026 or 2.6%. Gallup has rounded to the nearest whole percent, probably to make the margin of error easier to understand.

f. $(\text{lower bound}, \text{upper bound}) = (0.76 - 0.026, 0.76 + 0.026) = (0.734, 0.786)$

Alternate Solution

**Using the TI-83, 83+, 84, 84+ Calculators**

STAT TESTS A: 1-PropZinterval with $x = (0.76)(1005)$ (round up to 764),

$n = 1005$, $CL = 0.95$.

Answer is $(0.734, 0.787)$.

g. Samples help us approximate the true proportion we want to discover, but the sample proportion may not be precisely the same as that value. The margin of error allows us to create a range of reasonable values that we expect will capture the true proportion. We can say with 95% confidence that between 73% and 79% of all adult workers across the U.S. expect to continue working beyond retirement age. We are 95% confident, because 95% of the confidence intervals created from samples like this one will capture the true proportion.

Exercise 131.

_A national survey of 1,000 adults was conducted on May 13, 2013 by Rasmussen Reports. It concluded with 95% confidence that 49% to 55% of Americans believe that big-time college sports programs corrupt the process of higher education._

a. _Find the point estimate and the error bound for this confidence_
interval.

b. Can we (with 95% confidence) conclude that more than half of all American adults believe this?

c. Use the point estimate from part a and n = 1000 to calculate a 75% confidence interval for the proportion of American adults who believe that major college sports programs corrupt higher education.

d. Can we (with 75% confidence) conclude that at least half of all American adults believe this?

Solution

a. \( p' = \frac{(0.55 + 0.49)}{2} = 0.52; \text{EBP} = 0.55 - 0.52 = 0.03 \)

b. No, the confidence interval includes values less than or equal to 0.50. It is possible that less than half of the population believe this.

c. \( CL = 0.75, \text{so } \alpha = 1 - 0.75 = 0.25 \text{ and } \frac{\alpha}{2} = 0.125 \Rightarrow z = 1.150. \) (The area to the right of this \( z \) is 0.125, so the area to the left is \( 1 - 0.125 = 0.875. \))

\[ \text{EBP} = 1.150 \cdot \sqrt{\frac{0.52(0.48)}{1000}} \approx 0.018 \]

\( (p' - \text{EBP}, p' + \text{EBP}) = (0.52 - 0.018, 0.52 + 0.018) = (0.502, 0.538) \)

Alternate Solution

Using the TI-83, 83+, 84, 84+ Calculators:

STAT TESTS A: 1-PropZinterval with \( x = (0.52)(1000), n = 1000, CL = 0.75. \)

Answer is (0.502, 0.538)

d. Yes – this interval does not fall less than 0.50 so we can conclude that at least half of all American adults believe that major sports programs corrupt education – but we do so with only 75% confidence.
Exercise 132.

Public Policy Polling recently conducted a survey asking adults across the U.S. about music preferences. When asked, 80 of the 571 participants admitted that they have illegally downloaded music.

a. Create a 99% confidence interval for the true proportion of American adults who have illegally downloaded music.

b. This survey was conducted through automated telephone interviews on May 6 and 7, 2013. The error bound of the survey compensates for sampling error, or natural variability among samples. List some factors that could affect the survey’s outcome that are not covered by the margin of error.

c. Without performing any calculations, describe how the confidence interval would change if the confidence level changed from 99% to 90%.

Solution

\[ p' = \frac{80}{571} \approx 0.14 \quad q' = 1 - 0.14 = 0.86 \]

CL = 0.99, so \( \alpha = 1 - 0.99 = 0.01 \) and \( \alpha \frac{\alpha}{2} = 0.005 \)

\[ z_{\frac{\alpha}{2}} = 2.576 \quad (The \ area \ to \ the \ right \ of \ this \ z \ is \ 0.005, \ so \ the \ area \ to \ the \ left \ is \ 1 - 0.005 = 0.995.) \]

\[ EBP = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p'q'}{n}} = 2.576 \cdot \sqrt{\frac{0.14(0.86)}{571}} \approx 0.037 \]

Confidence interval: \( (0.14 - 0.037, 0.14 + 0.037) = (0.103, 0.177) \)

Alternate Solution:

**Using the TI-83, 83+, 84, 84+ Calculators**

Press STAT and arrow over to TESTS.

Arrow down to A:1-PropZint. Press ENTER.

Arrow down to \( x \) and enter 80.

Arrow down to \( n \) and enter 571.

Arrow down to C-Level and enter 0.99.
Arrow down to Calculate and press ENTER.

The confidence interval is (0.103, 0.178).

b. With phone surveys, researchers often have a difficult time reaching the participants selected. A large non-response rate can affect the reliability of the results. Also, many people are not comfortably admitting behavior that others might criticize. Many people may not have answered this question honestly. The true proportion of adults who have downloaded music illegally is probably greater than the survey shows.

d. When the confidence level decreases, the error bound for the confidence interval decreases as well. A 90% confidence interval will always be narrower than the 99% confidence interval for the same data.

Exercise 133. You plan to conduct a survey on your college campus to learn about the political awareness of students. You want to estimate the true proportion of college students on your campus who voted in the 2012 presidential election with 95% confidence and a margin of error no greater than five percent. How many students must you interview?

Solution

\[
CL = 0.95 \quad \alpha = 1 - 0.95 = 0.05 \quad \frac{\alpha}{2} = 0.025 \quad z_{\frac{\alpha}{2}} = 1.96
\]

Use \( p' = q' = 0.5 \).

\[
n = \frac{z_{\frac{\alpha}{2}}^2 p' q'}{EBP^2} = \frac{1.96^2(0.5)(0.5)}{0.05^2} = 384.16 \quad \text{You need to interview at least 385 students to estimate the proportion to within 5% at 95% confidence.}
\]

Exercise 134. In a recent Zogby International Poll, nine of 48 respondents rated the
likelihood of a terrorist attack in their community as “likely” or “very likely.” Use the plus-four method to create a 97% confidence interval for the proportion of American adults who believe that a terrorist attack in their community is likely or very likely. Explain what this confidence interval means in the context of the problem.

Let \( x = 9 + 2 = 11 \) and \( n = 48 + 4 = 52 \). Then \( \hat{p}' = \frac{11}{52} \sim 0.212 \).

CL = 0.97, so \( \alpha = 1 - 0.97 = 0.03 \) and \( \frac{\alpha}{2} = 0.015 \). \( z_{\frac{\alpha}{2}} = 2.17 \) (The area to the right of this \( z \) is 0.015, so the area to the left is \( 1 - 0.015 = 0.985 \).)

\[
EBP = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}'q'}{n}} = 2.17 \sqrt{\frac{0.212(0.788)}{52}} \sim 0.123
\]

\( (\hat{p}' - EBP, \hat{p}' + EBP) = (0.212 - 0.123, 0.212 + 0.123) \)

\( = (0.089, 0.335) \)

Alternate Solution

Using the TI-83, 83+, 84, 84+ Calculators:

STAT TESTS A: 1-PropZinterval with \( x = 11 \), \( n = 52 \), CL = 0.97.

Answer is (0.089, 0.334).

We are 97% confident that the true proportion of American adults that believe that a terrorist attack in their community is likely or very likely is between 8.9% and 33.5%.