Exam 2

For problems 1-6, circle the letter next to the response that BEST answers the question or completes the sentence. You do not have to show any work or write any explanations here. Make sure to read each statement carefully! 2 pts each

1. The probability of an event is always
   A) equal to 1
   B) between 0 and 1, inclusive
   C) between -1 and 1, inclusive
   D) between 0 and 100

2. The shaded area in the venn diagram on the right represents
   A) A and B
   B) A or B
   C) A / B
   D) the whole sample space

3. Two mutually exclusive events
   A) always occur together
   B) cannot occur together
   C) can sometimes occur together
   D) can occur together, provided one has already occurred

4. The number of ways one can select a four digit ATM code is
   A) 10^4
   B) 10!
   C) \( \binom{10}{4} \)
   D) \( 10 \times 10 \times 10 \times 10 \)

5. An ice-cream shop has 24 different flavors of ice-cream. How many ways can you select a two scoop ice-cream cone if you want two different flavors and the order they are placed doesn’t matters to you.
   A) 48
   B) 276
   C) 552
   D) 576
6. The number of ways one can select 6 people from a group of 15 to form a subcommittee is
   A) $6!$
   B) $15!$
   C) $\binom{15}{6}$
   D) $15^6$

For problems #7-13 you need to show work where possible and/or state which calculator program and
parameters you are using for your calculations. Unless otherwise stated, round final answers to four
decimal places.

7. The following probability distribution function shows the probabilities of owning $x$ number of cell
phones in a household in a certain town in Sweden. 6 pts

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>0.01</td>
<td>0.13</td>
<td>0.29</td>
<td>?</td>
<td>0.12</td>
<td>0.05</td>
</tr>
</tbody>
</table>

(a) Find the probability that a randomly picked household doesn’t own any cellphones

$$ P(0) = 0.01 $$

(b) Find the probability that a randomly picked family owns exactly 3 cell phones.

$$ P(3) = 1 - (0.01 + 0.13 + 0.29 + 0.12 + 0.05) = 0.40 $$

(c) Find the probability that a randomly picked family owns at least 4 cell phones.

$$ P(4) + P(5) = 0.12 + 0.05 = 0.17 $$

8. In a particular lottery one can win $5,000 or $100. A lottery ticket cost $2 and the probability
distribution for the net win (profit) is shown in the table below. 5 pts

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0.9989</td>
</tr>
<tr>
<td>98</td>
<td>0.001</td>
</tr>
<tr>
<td>9,998</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

$$ E(x) = \mu_x = \sum x \cdot P(x) $$

$$ = (-2)(0.9989) + (98)(0.001) + (9998)(0.0001) $$

$$ = -0.90 $$

We expect a loss of $0.90.
9. In a statistics class of 30 students, 19 uses the math tutoring center regularly. Suppose 2 people are randomly selected from this class, without replacement.

(a) Draw a tree diagram for this experiment, carefully labeling each branch of the tree with appropriate probabilities (in fraction form please). Let \( T = \text{student uses tutoring center} \)

(b) Let \( x \) represent the number of students out of those 2 selected that uses the math tutoring center regularly. Use the tree diagram to help complete the probability distribution table for \( x \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1264</td>
</tr>
<tr>
<td>1</td>
<td>0.4805</td>
</tr>
<tr>
<td>2</td>
<td>0.3931</td>
</tr>
</tbody>
</table>

\[
P(0) = \frac{11}{30} \cdot \frac{10}{29} = 0.1264 \\
P(1) = \frac{19}{30} \cdot \frac{11}{29} + \frac{11}{30} \cdot \frac{19}{29} = 0.4805 \\
P(2) = \frac{11}{30} \cdot \frac{19}{29} = 0.3931
\]

(c) Is this a binomial experiment? Explain why or why not.

No, because the events are not independent due to "without replacement."
10. The following contingency table gives the two-way classification of 226 people based on their gender and religious importance. *You can leave all answers below in fraction form.*

<table>
<thead>
<tr>
<th>Gender * Religious_Importance Crosstabulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="Image" alt="Contingency Table" /></td>
</tr>
</tbody>
</table>

(a) If you randomly select one person from the group above, find the probability that this selected person is fairly religious.

\[
\frac{99}{226} = 0.4381
\]

(b) If you randomly select one person from the group above, find the probability that this selected person is female *and* is very religious.

\[
\frac{39}{226} = 0.1726
\]

(c) If you randomly select one person from the group above, find the probability that this selected person male *or* is very religious.

\[
\frac{43 + 31 + 25 + 39}{226} = \frac{138}{226} \quad \text{(or} \quad \frac{99 + 64 - 25}{226})
\]

\[
= 0.6106
\]

(d) If you randomly select one person from the group above, find the probability that this selected person is very religious, *given* that it’s a female.

\[
\frac{39}{127} = 0.3071
\]

(e) Are the events "female" and "religion very important" independent? Support your answer using probabilities.

\[
P(F) \neq P(F|\text{Very}) \quad \text{or} \quad P(\text{Very}) \neq P(\text{Very}|F)
\]

\[
\frac{127}{226} = \frac{39}{64} \quad \text{Not independent b/c not equal}
\]

\[
\frac{64}{226} = \frac{39}{127} \quad \text{Independent b/c almost equal}
\]
11. Assume that a fair 20-sided dice is rolled. The sample space for one roll is \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}. 

(a) Find the probability of rolling a 15 when rolling the dice once.
\[
\frac{1}{20} = 0.05
\]

(b) Find the probability of rolling a number greater than 15 when rolling the dice once.
\[
\frac{5}{20} = \frac{1}{4} = 0.25
\]

(c) If the die was rolled 5 times, how big would the sample space be? That is, how many possible outcomes are there?
\[
20 \cdot 20 \cdot 20 \cdot 20 \cdot 20 = 20^5 = 3,200,000
\]

(d) Find the probability of rolling all 15’s if the dice is rolled 5 times. Write in fraction form or include at least 10 decimal places.
\[
\frac{1}{20} \cdot \frac{1}{20} \cdot \frac{1}{20} \cdot \frac{1}{20} \cdot \frac{1}{20} = \frac{1}{(20)^5} = \frac{1}{3,200,000} = 0.0000003125
\]

12. Jose has six chemistry books, three history books, and eight statistics books. 

a) He wants to choose one book from each subject to study. In how many ways can he choose the three books?
\[
6 \cdot 3 \cdot 8 = 144
\]

b) Once he has chosen the three books, he wants to put them on a special shelf. How many different ways could he arrange (order) these three books on his shelf?
\[
3! = 3 \cdot 2 \cdot 1 = 6
\]
13. Individual plays on a slot machine are independent. The probability of winning on any play is 0.38.

(a) What is the probability of winning 3 times in a row?

\[
(0.38)(0.38)(0.38) = 0.0549
\]

(b) Suppose the slot machine is played 15 times in a row. What is the probability of winning exactly 10 times?

\[X \sim B(15, 0.38)\] where \(X = \# \text{ of times of winning}\)

\[P(X=10) = 0.0173\]

(c) Suppose the slot machine is played 15 times in a row. What is the probability of winning at least 10 times?

\[P(X \geq 10) = 0.0232\]