Exam 3

For problems 1-8, circle the letter next to the response that best answers the question or completes the sentence. You do not have to show any work or write any explanations here. Make sure to read each statement carefully! (2 pts each)

1. The following figure is a probability density curve that represents the lifetime, in months, of a certain type of laptop battery. Find the probability that a randomly selected battery will have a lifetime of more than 10 months.
   \[\text{A)} \quad 0.86 \quad \text{B)} \quad 0.43 \quad \text{C)} \quad 0.29 \quad \text{D)} \quad 0.14\]

2. Which of the following is not true of the normal distribution?
   A) The total area under the curve is 1.0
   B) The mean is always zero
   C) The curve is bell-shaped
   D) The two tails of the curve extend indefinitely

3. A confidence interval is an interval that is used to estimate a:
   A) sample statistic based on the information from a population
   B) sample statistic based on the information from a sample
   C) population parameter based on the information from a population
   D) population parameter based on the information from a sample

4. A 95% confidence interval for \( \mu \) can be interpreted to mean that if we take 100 samples of the same size and construct 100 such confidence intervals for \( \mu \), then
   A) 95 of them will include \( \bar{x} \)
   B) 95 of them will not include \( \bar{x} \)
   C) 95 of them will include \( \mu \)
   D) 95 of them will not include \( \mu \)
5. The width of a confidence interval depends on the size of the:
A) point estimate  
B) margin of error  
C) sample mean  
D) population mean

6. The three confidence intervals below were constructed from the same sample. One of them was computed at a confidence level of 90%, another at a confidence level of 95%, and another at a confidence level of 98%. Which is the confidence level at 90%?
A) $23.3 < \mu < 30.7$  
B) $22.6 < \mu < 31.4$  
C) $21.8 < \mu < 32.2$  
D) cannot be determined

7. A scientist is estimating the mean lifetime of a newly-discovered insect. From a sample of 100 insects, she finds a sample mean of 48.8 days and a sample standard deviation of 2.725 days. Which calculator program should she use if she wants to create a confidence interval to estimate the mean lifetime of all such insects?
A) ZInterval...  
B) TInterval...  
C) 1-PropZInt...  
D) We can't use any of those as we don't know the distribution of our population

8. The following stem-and-leaf plots each illustrate a sample. Which sample can be treated as approximately normal?

A) 

B) 

C) 

D) 

Circle the two correct words that completes the sentence below

9. By increasing the sample size when creating a confidence interval, you can increase/decrease the confidence level or you can make the interval narrower/wider.
For the rest of the problems, you have to show work!
For all problems involving a normal or t-distribution, you must draw a picture corresponding to the problem, with correctly labeled axis, and shaded region(s). You also need to state what calculator program you are using for your calculations. All probabilities need to be rounded to four decimal places.
When finding confidence intervals, you must do it by hand and show all 5 steps!

10. a) Find \( P(t > -1.7) \) if \( df = 8 \) 

b) Find \( P(z < 2.2) \) 

\[
\begin{array}{c}
\text{Normal} 8.3 \\
\text{Option 2} \\
\text{mean} = 0 \\
\text{SD} = 1 \\
B = 2.2
\end{array}
\]

\[
0 = 1 \\
0.9861 \\
2.2 \\
z
\]

11. In January of 2015 it was reported that 16-24 year olds spend an average of 2.68 hours per day on social networking. Suppose these times have a normal distribution and that the standard deviation is 0.9 hours.

a) What is the probability of a randomly picked person in this age group spending more than 4.5 hours per day social networking? 

\[
\begin{array}{c}
\text{Normal} 8.3 \\
\text{Option 1} \\
\text{mean} = 2.68 \\
\text{SD} = 4.5
\end{array}
\]

\[
P(x > 4.5) = 0.0216
\]

b) Find the 80th percentile of this population.
b) If a random sample of 13 people in this age group were selected, what is the probability that the mean number of hours spent social networking among these people is between 1.5 and 2.5 hours? (4 pts)

\[ P(1.5 < \bar{x} < 2.5) = 0.2354 \]

Standard Normal Distribution

Normal Distribution

12. A researcher took a simple random sample of 27 judges and asked them whether or not they favor the death penalty. 16 of the judges answered "yes", and 11 of them answered "no".

Based on this sample, construct 98% confidence interval for the proportion of all judges who are in favor of the death penalty. Round final answers to four decimal places. (11 pts)

1. point estimate:

\[ \hat{p} = \frac{16}{27} \approx 0.5926 \]

2. critical value:

\[ z = 2.3263 \]

3. standard error and margin of error:

Standard error: \[ \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.5926(1-0.5926)}{27}} = 0.09456 \]

Margin of error: \[ z \cdot \sigma_{\hat{p}} = (2.3263)(0.09456) = 0.2200 \text{ (or 0.21997)} \]

4. confidence interval:

\[ \hat{p} \pm m = 0.5926 \pm 0.21997 = (0.3726, 0.8126) \]

5. full sentence interpretation of result:

We are 98% confident that the true proportion of all judges who favor the death penalty is between 0.3726 and 0.8126.
13. A company randomly selected nine office employees and secretly monitored their computers for one month. The times (in hours) spent by these employees using their computers for non-job-related activities (playing games, personal communications, social media etc.) during this month are given below.

\[ 7 \quad 12 \quad 9 \quad 8 \quad 11 \quad 4 \quad 16 \quad 2 \quad 6 \]

\[ \bar{x} = 8.3 \quad \text{Var}(x) = 4.2720 \]

a) Assuming that such times for all employees are normally distributed, construct a 95% confidence interval for the corresponding population mean for all employees of this company.

*Round final answers to two decimal places.*

1. **Point estimate:**

\[ \bar{x} = 8.3 \]

2. **Critical value:**

\[ T_{\text{invsig 83}} = 2.306 \]

3. **Standard error and margin of error:**

\[ s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{4.2720}{\sqrt{9}} = 1.424 \]

\[ m = t \cdot s_{\bar{x}} = (2.306)(1.424) = 3.2837 \]

4. **Confidence interval:**

\[ CI: x \pm m = 8.3 \pm 3.2837 = (5.05, 11.62) \]

5. **Full sentence interpretation of result:**

We are 95% confident that the mean time spent on non-job-related activities on work computer per month among all employees at this company is between 5.05 and 11.62 hrs.

b) Suppose you needed to check whether this sample indeed came from an approximately normal population distribution. Please make a normal probability plot (also called normal quantile plot) of the sample on your calculator. Then sketch the plot in the window to the right.

(2 pts)

c) Based on your plot in part b), does this sample seem to come from an approximately normal distribution? **Explain** why or why not!

Yes, since it resembles a diagonal line.

(2 pts)
14. Based on a simple random sample, we are 98% confident that the proportion of people of Mexican heritage in Watsonville is between 0.715 and 0.811.

(a) What was the sample proportion?

\[ \hat{p} = \frac{0.715 + 0.811}{2} = 0.763 \]

(b) What is the margin of error?

\[ m = 0.811 - 0.763 = 0.048 \]

15. A store owner wants to estimate at a 97% confidence level the mean amount spent by all customers at his store. The manager knows that the standard deviation of amounts spent by all customers at his store is $26. What sample size should he choose so that the estimate has a margin of error of $5?

\[ n = \left( \frac{Z \cdot \sigma}{m} \right)^2 \]

\[ n = \left[ \frac{(2.1701)(26)}{5} \right]^2 \]

\[ n \approx 127.34 \]

\[ n \geq 128 \]

He should choose a sample size of at least 128 customers.

It is crucial that you DO NOT MISS CLASS in the coming week, as we will introduce the very important topic of hypothesis testing.

We will also do an in-class lab tomorrow, where you can earn 5 pts by being there on time and participating throughout the whole class period.
Exam 3

For problems 1-8, circle the letter next to the response that best answers the question or completes the sentence. You do not have to show any work or write any explanations here. Make sure to read each statement carefully! (2 pts each)

1. The following figure is a probability density curve (function) that represents the lifetime, in months, of a certain type of laptop battery. Find the probability that randomly selected battery will have a lifetime of more than 16 months.

   A) 0.14  
   B) 0.29  
   C) 0.43  
   D) 0.57 \( \left( 1 - 0.14 - 0.29 \right) \)

2. Which of the following is not true of the normal distribution?
   A) The total area under the curve is 1.0
   B) The two tails of the curve extend indefinitely
   C) The curve is bell-shaped
   D) The mean is always zero

3. A confidence interval is an interval that is used to estimate a:
   A) population parameter based on the information from a sample
   B) sample statistic based on the information from a sample
   C) population parameter based on the information from a population
   D) sample statistic based on the information from a population

4. A 95% confidence interval for \( \mu \) can be interpreted to mean that if we take 100 samples of the same size and construct 100 such confidence intervals for \( \mu \), then
   A) 95 of them will include \( \mu \)
   B) 95 of them will not include \( \mu \)
   C) 95 of them will include \( \bar{x} \)
   D) 95 of them will not include \( \bar{x} \)
5. The width of a confidence interval depends on the size of the:
   A) sample mean
   B) population mean
   C) point estimate
   D) margin of error

6. The three confidence intervals below were constructed from the same sample. One of them was computed at a confidence level of 90%, another at a confidence level of 95%, and another at a confidence level of 98%. Which is the confidence level at 90%?
   A) $21.8 < \mu < 32.2$
   B) $22.6 < \mu < 31.4$
   C) $23.3 < \mu < 30.7$
   D) cannot be determined

7. A scientist is estimating the mean lifetime of a newly-discovered insect. From a sample of 100 insects, she finds a sample mean of 48.8 days. Suppose that the population standard deviation of all lifetimes is 2.725 days. Which calculator program should she use if she wants to create a confidence interval to estimate the mean lifetime of all such insects?
   A) ZInterval...
   B) TInterval...
   C) 1-PropZInt...
   D) We can't use any of those as we don't know the distribution of our population

8. The following stem-and-leaf plots each illustrate a sample. Which sample can be treated as approximately normal?
   A) B) C) D)

   | 0 | 2367 |
   | 1 | 111236789 |
   | 2 | 01280 |
   | 3 | 1 |
   | 4 | 02 |
   | 5 | 1 |
   | 6 | 7 047 |
   | 7 | 8 0133799 |
   | 8 | 9 23355678 |

   | 2 | 9 |
   | 3 | 2 |
   | 4 | 3 |
   | 5 | 47 |
   | 6 | 29 |
   | 7 | 047 |
   | 8 | 0133799 |
   | 9 | 23355678 |

   | 0 | 133569 |
   | 1 | 08 |
   | 2 | 11 |
   | 3 | 0 |
   | 4 | 3 |
   | 5 | 113 |
   | 6 | 22245679 |
   | 7 | 2158 |
   | 8 | 3 |

   | 1 | 7 |
   | 2 | 6 |
   | 3 | 68 |
   | 4 | 33344 |
   | 5 | 113 |
   | 6 | 22245679 |
   | 7 | 2158 |
   | 8 | 3 |

   Circle the two correct words that completes the sentence below

9. You can increase the confidence level of a confidence interval by making the interval wider or by increasing the sample size. (2 pts)
For the rest of the problems, you have to show work!
For all problems involving a normal or t-distribution, you must draw a picture corresponding to the problem, with correctly labeled axis, and shaded region(s). You also need to state what calculator program you are using for your calculations. All probabilities need to be rounded to four decimal places.
When finding confidence intervals, you must show and number all 5 steps!

10. a) Find \( P(t > 1.7) \) if \( df = 8 \) 

Problem skipped as not all students had T83 on calculator (and I told them they didn't need it.)

\[ P(t > 1.2) = 0.8849 \] (3 pts)

b) Find \( P(Z > -1.2) \)

\[ \text{Normal 83} \]
\[ \sigma = 1 \]
\[ \text{Ophm} = 1 \]
\[ \text{Mean} = 0 \]
\[ \text{SD} = 1 \]
\[ A = -1.2 \]

11. In January of 2015 it was reported that 16-24 year olds spend an average of 2.68 hours per day on social networking. Suppose these times have a normal distribution and that the standard deviation is 0.7 hours.

a) What is the probability of a randomly picked person in this age group spending more than 4.0 hours per day social networking?

\[ \text{Normal 83} \]
\[ \sigma = 0.7 \]
\[ \text{Ophm} = 1 \]
\[ \text{Mean} = 2.68 \]
\[ \text{SD} = 0.7 \]
\[ A = 4 \]
\[ P(x > 4) = 0.0297 \] (3 pts)

b) Find the 30th percentile of this population.

\[ \text{Invnorm 83} \]
\[ \text{left tail area} = 0.30 \]
\[ \text{Mean} = 2.68 \]
\[ \text{SD} = 0.7 \]
\[ x = 2.3129 \]
c) If a random sample of 10 people in this age group were selected, what is the probability that the mean number of hours spent social networking among these people is between 2.0 and 2.5 hours? (4 pts)

\[
\begin{align*}
\bar{x} &= \frac{0.7}{\sqrt{10}} \\
\sigma &= 0.2214 \\
P(2.0 < \bar{x} < 2.5) &= 0.207
\end{align*}
\]

12. A researcher took a simple random sample of 27 judges and asked them whether or not they favor the death penalty. 16 of the judges answered “yes”, and 11 of them answered “no”.

Based on this sample, construct 98% confidence interval for the proportion of all judges who are in favor of the death penalty. Round final answers to four decimal places. (12 pts)

1. Point estimate:
\[ \hat{p} = \frac{16}{27} \approx 0.5926 \]

2. Critical value:
\[ \alpha = 0.02 \]
\[ z = 2.3263 \]

3. Standard error and margin of error:
\[ \text{Standard error: } \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.5926(1-0.5926)}{27}} = 0.09456 \]
\[ \text{Margin of error: } z \cdot \hat{p} = (2.3263)(0.09456) = 0.2200 \text{ (or 0.21997)} \]

4. Confidence interval:
\[ \hat{p} \pm m = 0.5926 \pm 0.21997 = (0.3726, 0.8126) \]

5. Full sentence interpretation of result:
We are 98% confident that the true proportion of all judges who favor the death penalty is between 0.3726 and 0.8126.
13. A company randomly selected nine office employees and secretly monitored their computers for one month. The times (in hours) spent by these employees using their computers for non-job-related activities (playing games, personal communications, social media etc.) during this month are given below.

\[ 7 \ 12 \ 9 \ 8 \ 11 \ 4 \ 16 \ 2 \ 6 \]

\[ \bar{x} = 8.3 \quad s = 4.2720 \]

a) Assuming that such times for all employees are normally distributed, construct a 95\% confidence interval for the corresponding population mean for all employees of this company. Round final answers to two decimal places. (12 pts)

\[ \text{1. point estimate: } \bar{x} = 8.3 \]

\[ \text{2. critical value: } t_{0.95} = 2.306 \]

\[ \text{3. standard error and margin of error: } s_x = \frac{s}{\sqrt{n}} = \frac{4.2720}{\sqrt{9}} = 1.424 \]

\[ m = t \cdot s_x = (2.306)(1.424) = 3.2837 \]

\[ \text{4. confidence interval: } \bar{x} \pm m = 8.3 \pm 3.2837 = (5.02, 11.62) \]

\[ \text{5. full sentence interpretation of result: } \]

We are 95\% confident that the mean time spent on non-job-related activities on work computer per month among all employees at this company is between 5.02 and 11.62 hrs.

b) Suppose you needed to check whether this sample indeed came from an approximately normal population distribution. Please make a normal probability plot (also called normal quantile plot) of the sample on your calculator. Then sketch the plot in the window to the right. (2 pts)

\[ \text{normal probability plot} \]

c) Based on your plot in part b), does this sample seem to come from an approximately normal distribution? Explain why or why not! (2 pts)

Yes, since it resembles a diagonal line.
14. Based on a simple random sample, we are 98% confident that the proportion of people that are in the age group 20 to 29 in Watsonville is between 0.141 and 0.183.

\[
\hat{p} = \frac{0.141 + 0.183}{2} = 0.162
\]

a) What was the sample proportion?

\[
\hat{p} = \frac{0.141 + 0.183}{2} = 0.162
\]

b) What is the margin of error?

\[
m = 0.183 - 0.162 = 0.021
\]

15. A store owner wants to estimate at a 99% confidence level the mean amount spent by all customers at his store. The manager knows that the standard deviation of amounts spent by all customers at his store is $26. What sample size should he choose so that the estimate has a margin of error of $3?

\[
\alpha = 0.01 \quad \alpha/2 = 0.005
\]

\[
\begin{align*}
2 & \quad n = \left(\frac{2.5758}{26}\right)^2 \\
4 & \quad n = \left(\frac{22.3236}{3}\right)^2 \\
1 & \quad n \approx 499 \text{ (rounded up)}
\end{align*}
\]

He should choose a sample size of at least 499 customers.

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We will also do an in-class lab tomorrow, where you can earn 5 pts by being there on time and participating throughout the whole class period.