Exam 2

You are allowed to use a 3" x 5" note card (one side) and a scientific calculator. You must show work to support your answers. A correct answer without supporting work will not receive full credit. Simplify all your answers and indicate your answers clearly.

Do the problems that you find to be easy first. That way you build your confidence and you are more likely to successfully solve the more challenging problems. You have 1 hour and 15 minutes to finish the exam. Good luck!

1. Determine if each of the following statements are true T or false F. Read the statements carefully! You do not have to show work here. (12 pts)

   (a) The inequality $|3x+5| > -2$ has no solution ______ many solutions ______
       ____________
       ________
       ________

   (b) The equation $|3x+5| = -2$ has no solution

   (c) The compound inequality $x > 5$ AND $x \leq -3$ has no solution

   (d) The expression $(x^2 + 5)$ is called a monomial binomial

   (e) The solutions to $6x^2 - 13x + 6 = 0$ corresponds to the x-intercepts of the graph of $f(x) = 6x^2 - 13x + 6$

   (f) $x^2 + 36$ is prime

2. Solve the inequality. Other than $\emptyset$ graph the solution set on a number line. (5 pts)

   \[
   2x + 3 \leq 3(2x + 4) \\
   2x + 3 \leq 6x + 12 \\
   -4x \leq 9 \\
   \frac{-4x}{-4} \geq \frac{9}{-4} \\
   x \geq \frac{-9}{4} \\
   \text{or} \quad x \geq -2.25
   \]
3. Solve the compound inequality. Express the solution set in interval notation.

\[-20 < 3x + 1 \leq 4\]

\[
\begin{array}{c}
-1 \\
\hline
-1 \\
\hline
-\frac{21}{3} \leq 3x \leq \frac{3}{3} \\
\hline
-7 \leq x \leq 1
\end{array}
\]

\[(-7, 1]\]

4. Solve the compound inequalities. Express the solution sets in interval notation.

(a) \[x - 3 > 7 \text{ OR } 4x + 2 \geq -6\]

\[
\begin{array}{c}
+3 +3 \\
\hline
x > 10
\end{array}
\quad
\begin{array}{c}
-2 -2 \\
\hline
4x \geq -8 \\
\hline
x \geq -2
\end{array}
\]

\[[-2, \infty)\]

(b) \[x - 3 > 7 \text{ AND } 4x + 2 \geq -6\] (note: inequalities are the same as in (a), but now with AND instead of OR, so you don't need to re-solve them)

\[(10, \infty)\]

5. Solve the equation

\[2|x-3| - 7 = 9\]

\[
\begin{array}{c}
+7 +7 \\
\hline
2|x-3| = 16 \\
\hline
\frac{2}{2} \\
|x-3| = 8
\end{array}
\]

\[x-3 = 8 \quad \text{or} \quad x-3 = -8\]

\[
\begin{array}{c}
+3 +3 \\
\hline
x = 11
\end{array}
\quad
\begin{array}{c}
+3 +3 \\
\hline
x = -5
\end{array}
\]

\[\{-5, 11\}\]
6. Solve the inequality. Express your answer in **interval notation**.

\[ |x - 2| + 3 > 5 \]
\[ -3 - 3 \]
\[ |x - 2| > 2 \]

The distance of \( x - 2 \) from 0 is greater than 2.

\[ \rightarrow \]
\[ -2 \quad 0 \quad 2 \]

\[ \frac{x - 2 < -2}{+2} \quad \frac{x - 2 > 2}{+2} \]
\[ x < 0 \quad x > 4 \]

\[ (-\infty, 0) \cup (4, \infty) \]

(6 pts)

7. Solve the inequality. Express your answer in **interval notation**.

\[ |x| \leq 4 \]

The distance of \( x \) from 0 is less than or equal to 4.

\[ \rightarrow \]
\[ -4 \quad 0 \quad 4 \]

\[ [-4, 4] \]

(4 pts)

8. Graph the system of inequalities in the coordinate system provided. Make sure to make it very clear where your solutions are located.

\[ \begin{align*}
  x + y < 4 \\
  y \geq 2x - 4
\end{align*} \]

\[ y < -x + 4 \]

\[ \text{slope} = -1 \]

\[ \text{Shade} \quad \text{above} \]

\[ \text{Solid line} \]

\[ \text{Dashed line} \]

(5 pts)
For problems 9 to 12, factor completely or state that it is prime

9. \[32y^2 + 48y + 18\]
   \[= 2(16y^2 + 24y + 9)\]
   \[= 2(4y + 3)^2\]

10. \[x^3 - 2x^2 - x + 2\]
    \[= x(x-2) - 1(x-2)\]
    \[= (x-2)(x^2 - 1)\]
    \[= (x-2)(x-1)(x+1)\]

11. \[x + 8x^4\]
    \[= x(1 + 8x^3)\]
    \[= x((1)^3 + (2x)^3)\]
    \[= x(1 + 2x)(1 - 2x + (2x)^2)\]
    \[= x(1 + 2x)(1 - 2x + 4x^2)\]

12. \[y^8 - 8y^4 + 15\]
    Let \(u = y^4\)
    \[= (u^2 - 8u + 15)\]
    \[= (u - 5)(u - 3)\]
    \[= (y^4 - 5)(y^4 - 3)\]
13. Use factoring to solve for $x$.

\[ 6x^2 + 5x = 4 \]
\[
\begin{align*}
\text{ac} &= 6 (-4) = -24 \\
8 &= -3
\end{align*}
\]
\[ (3x+4) = 0 \quad \text{or} \quad (2x-1) = 0 \]
\[ x = \frac{-4}{3} \quad \text{and} \quad x = \frac{1}{2} \]

14. Find the x-intercepts of the function $f(x) = x^2 - 6x$. Write the intercepts as ordered pairs of the form $(x, y)$.

To find $x$-int, set $y = 0$ (or $f(x) = 0$)

\[ x^2 - 6x = 0 \]
\[ x(x - 6) = 0 \]
\[ x = 0 \quad \text{or} \quad x - 6 = 0 \]
\[ x = 6 \]

$x$-int: $(0, 0)$ and $(6, 0)$

15. Find the lengths of the three sides of the right triangle in the figure shown. Solve your equation using the factoring method.

\[ a^2 + b^2 = c^2 \]
\[ (x+7)^2 + x^2 = (x+8)^2 \]
\[ x^2 + 14x + 49 + x^2 = x^2 + 16x + 64 \]
\[ -16x - 64 \]
\[ x^2 - 2x - 15 = 0 \]
\[ (x - 5)(x + 3) = 0 \]
\[ x - 5 = 0 \quad \text{or} \quad x + 3 = 0 \]
\[ x = 5 \quad \text{or} \quad x = -3 \]
\[ x = 5 \quad \text{and} \quad x = -3 \]

The three sides are 5, 12, and 13 units long.
16. Match the following equations with correct graph, using your knowledge of whether or not a graph is a polynomial and the leading coefficient test. (6 pts)

(a) \( f(x) = -\frac{1}{20} x^2 + \frac{1}{5} x - \frac{1}{5} \) corresponds to graph E

(b) \( g(x) = \frac{1}{8} x^4 - \frac{1}{8} x^3 - \frac{17}{8} x^2 + \frac{21}{8} x + \frac{9}{2} \) corresponds to graph C

(c) \( h(x) = -x^3 + 12x^2 - 48x + 64 \) corresponds to graph A

Extra Credit: Factor \( 9x^2 - y^4 + 25 - 30x \)

\[
= \frac{9x^2 - 30x + 25 - y^4}{4}
= (3x - 5)^2 - (y^2)^2
= (3x - 5 + y^2)(3x - 5 - y^2)
\] (5 pts)