Simplify the following expressions as completely as possible or state that it’s not a real number. Rationalize the denominator if necessary. All variables represent positive numbers.

1. \(-\frac{\sqrt{16}}{\sqrt{49}}\)

2. \(\sqrt{15x^2} \cdot \sqrt{3x}\)

3. \(\sqrt[3]{64}\)

4. \(\frac{\sqrt{75x^3}}{\sqrt{3x}}\)

5. \(\sqrt{4} - 7\)

6. \(\sqrt{49y} \cdot \sqrt{36y^8}\)

7. \(\sqrt{-32}\)
8. \(-\sqrt{0.36} + \sqrt{0}\)

9. \(\frac{\sqrt{400x^{40}}}{\sqrt{2x}}\)

10. Without using a calculator, determine which two integers the number \(\sqrt{70}\) fall in-between? Explain or show how you figured it out.

Simplify the rational expression

11. \(\frac{x-1}{4x^2 + 3x - 7}\)

Perform the indicated operations and simplify completely. (Check your factoring carefully!)

12. \(\frac{x^3 + 5x + 6}{(x+2)^2} \cdot \frac{x + 2}{x + 3}\)

13. \(\frac{5(x+6)}{2-x} \div \frac{6(x^2 - 36)}{x^3 - 8x + 12}\)
14. \(\frac{4x-9}{3x-4} - \frac{x-5}{3x-4}\)

15. \(\frac{2}{4x-12} + \frac{4x+5}{2x^2-12x+18}\)

For what value(s) of \(x\) is the following expression undefined and why?

16. \(\frac{x^5-3x^4+4x^3+32x+1}{x^2+7x-18}\)

Solve the equation

17. \(\frac{2x}{3} - \frac{1}{6} = 2x\)

18. Given the equation: \(\frac{3}{x^2-1} + 1 = \frac{2}{x-1}\)

(a) For what value(s) of \(x\) is the equation undefined?

(b) Solve the equation.
Express the perimeter of the rectangle as a single rational expression

19.
\[ \frac{x}{x+1} \]
\[ \frac{x}{x-1} \]

20. A painter can paint a fence around a house in 6 hours. Working alone, the painter’s apprentice can paint the same fence in 12 hours. How many hours would it take them to do the job if they worked together?

21. A boat travels 5 km upstream in the same amount of time that the boat covers 15 km downstream. The current in the stream moves at a speed of 2 km/h. What is the speed of the boat in still water? (Don’t forget to define your variables, set up an equation, and write a full sentence answer. You might want to set up a table.)

22. Simplify \((3+\sqrt{2})(5+\sqrt{2})\)

23. Solve \(\sqrt{2x-1} + 2 = x\)