Exam 2

You are allowed to use a 3" x 5" note card and a scientific calculator. Read the instructions/questions carefully. You must show work to support your answers. A correct answer without supporting work will not receive full credit. Simplify all your answers and indicate your answers clearly (you will not receive full credit if leaving more than one answer).

Unless labeled otherwise, we will assume that every tick mark on the axes of the coordinate systems on this exam represents one unit.

1. Find at least 3 ordered pair solutions of the equation, then plot the points, and connect the points with a line. (6 pts)

\[ y = \frac{2}{3}x - 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y = \frac{2}{3}x - 1</th>
<th>(x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = \frac{2}{3}(0) - 1 = -1 )</td>
<td>(0,-1)</td>
</tr>
<tr>
<td>3</td>
<td>( y = \frac{2}{3}(3) - 1 = 1 )</td>
<td>(3,1)</td>
</tr>
<tr>
<td>-3</td>
<td>( y = \frac{2}{3}(-3) - 1 = -3 )</td>
<td>(-3,-3)</td>
</tr>
</tbody>
</table>

2. Determine whether \( \left( \frac{4}{3}, -1 \right) \) is a solution to the equation \( 3x - 2y = 6 \) (6 pts)

\[
3 \left( \frac{4}{3} \right) - 2(-1) = 6
\]

\[
4 + 2 = 6
\]

\( 6 = 6 \) \( \checkmark \)

Yes, \( \left( \frac{4}{3}, -1 \right) \) is a solution.
3. Graph the line that goes through the point \((3, -1)\) and that has slope \(\frac{-2}{1} = \frac{2}{-1}\). Plot at least 3 points. (5 pts)

4. Without graphing the equation, find the x-intercept and the y-intercept of the line represented by the following equation. (Remember to write the intercepts as points.) (6 pts)

\[7 - 3x = 13y\]

x-intercept: \((\frac{7}{3}, 0)\)
y-intercept: \((0, \frac{7}{13})\)

5. Solve the equation for \(y\), and then determine the slope and y-intercept of the line. (5 pts)

\[\frac{4x}{9} = \frac{6y}{9} + \frac{9}{9}\]

\[\frac{\frac{4x}{9} - \frac{9}{9}}{\frac{6}{9}} = \frac{6y}{9}\]

\[\frac{\frac{2}{3}x - \frac{3}{2}}{\frac{2}{3}} = y\]

slope: \(\frac{2}{3}\)
y-intercept: \((0, -\frac{3}{2})\)
6. Write the equation of the line which goes through the point \((4, -6)\) and has slope \(5\), in slope-intercept form. Hint: First use the point-slope form.

\[
\begin{align*}
y - y_1 &= m(x - x_1) \\
y - (-6) &= 5(x - 4) \\
y + 6 &= 5x - 20 \\
y - 6 &= 5x - 26 \\
\underline{\hspace{2cm}} \\
y &= 5x - 26
\end{align*}
\]

(7 pts)

7. Find the equations for the following lines.

(a) \(y = 2\)

(8 pts)

(b) \(y = \frac{4}{7}x + (-4)\)

\[
\begin{align*}
\underline{\text{or}} \\
y &= \frac{4}{7}x - 4
\end{align*}
\]
8. A car is depreciating in value every year. My friend Bob's 2 year old car is worth $21,400. He knows that when it's 6 years old it will be worth only $6,800. We can look at this information as two ordered pairs: (2, 21400) and (6, 6800).

(a) Find the slope of the line that goes through the ordered pairs (2, 21400) and (6, 6800).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6800 - 21400}{6 - 2} = \frac{-14600}{4} = -3650 \]

(b) What does the slope represent in the context of this problem?

The value of Bob's car is decreasing by $3650 per year.

(c) Find an equation that represents the car's value, y, in dollars, after x years.

\[ y - y_1 = m (x - x_1) \]
\[ y - 21400 = -3650 (x - 6) \]
\[ y - 6800 = -3650x + 21900 \]
\[ y + 6800 = -3650x + 28700 \]
\[ y = -3650x + 28700 \]

(d) Find the y-intercept and then interpret what it means in the context of this problem.

\[ y - \text{int.} = (0, 28700) \]

The price of Bob's car when new, was $28,700.

9. Solve the following system of equations by graphing. Show how you go about graphing the equations. Please write your solution as a point.

\[ \begin{align*}
2x + 2y &= 2 \\
-x + y &= 2 \\
\end{align*} \]
\[ \begin{align*}
-2x &= -2 \\
y &= -1 \frac{1}{2} x + 1 \\
\end{align*} \]
\[ y = x - 2 \]
\[ (2, 0) \]
10. Solve the system of equations by the substitution method. If there is one solution, check your solution. If there is no solution or infinitely many solutions, so state. (8 pts)

\[
\begin{align*}
-x + 3y &= 10 \\
2x + 8y &= -6
\end{align*}
\]

\[
\begin{align*}
x &= 10 - 3y \\
\frac{-x}{-1} &= \frac{10 - 3y}{-1} \\
-x &= -10 + 3y \\
x &= 3y - 10
\end{align*}
\]

\[
2(3y - 10) + 8y = -6
\]

\[
\begin{align*}
6y - 20 + 8y &= -6 \\
14y &= -26 \\
y &= \frac{-26}{14} \\
y &= -2
\end{align*}
\]

\[
\begin{align*}
(x, y) &= (3y - 10, -2) \\
(x, y) &= (3(-2) - 10, -2) \\
(x, y) &= (-7, -2)
\end{align*}
\]

Check:

\[
\begin{align*}
-(-7) + 3(1) &= 10 \\
7 + 3 &= 10 \\
10 &= 10 \checkmark
\end{align*}
\]

\[
\begin{align*}
2(-7) + 8(1) &= -6 \\
-14 + 8 &= -6 \\
-6 &= -6 \checkmark
\end{align*}
\]

11. Solve the system of equations by the addition method. If there is one solution, check your solution. If there is no solution or infinitely many solutions, so state. (7 pts)

\[
\begin{align*}
3 \cdot \begin{cases} 2x + 4y &= 5 \\
3x + 6y &= 6
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
6x + 12y = 15 \\
-6x - 12y = -12
\end{array} \right. \\
\hline
0 = 3
\end{align*}
\]

False

No Solution
12. Clearly define **two variables** and write a **system of equations** describing the given conditions. Solve the system using method of your choice and answer the questions using a whole sentence with correct units. (10 pts)

At my favorite taqueria I can buy 5 tacos and 2 burritos for $19.00, or 3 tacos and 3 burritos for $18.15. What is the price for one taco and one burrito respectively?

Let \( x = \text{the price for one taco} \)
Let \( y = \text{the price for one burrito} \)

\[ \begin{align*}
3. & \begin{cases}
5x + 2y = 19 \\
-2x + 3y = 18.15
\end{cases} \\
\Rightarrow & \begin{cases}
5(2.3) + 2y = 19 \\
11.5 + 2y = 19
\end{cases}
\Rightarrow 2y = 7.5 \\
\frac{2y}{2} = \frac{7.5}{2} \\
y = 3.75
\end{align*} \]

\[ \begin{align*}
9x & = 20.7 \\
x & = 2.3
\end{align*} \]

13. Determine if the following statements are True or False: (12 pts)

(a) The point \((0, 0)\) is also called the origin \( T \)
(b) Two lines with the slopes -2 and 2 respectively are perpendicular \( F \)
(c) If two distinct lines have the same slope they must be parallel \( T \)
(d) The graph \( x = 3 \) is a vertical line \( T \)
(e) Vertical lines have an undefined slope \( T \)
(f) Horizontal lines have slope = 0 \( T \)
(g) The line represented by the equation \( y = x + 5 \) has slope = 0 \( F \)
(h) The line represented by the equation \( y = 3x \) has its y-intercept at \((0, 0)\). \( T \)
(i) The system \( \begin{cases}
y = 2x - 8 \\
y = 2x + 5
\end{cases} \) has infinitely many solutions \( F \)
(j) The system \( \begin{cases}
y = 9x - 8 \\
y = -9x - 8
\end{cases} \) has exactly one solution \( T \)
(k) \( x - 6y = 4 \) is a linear equation \( T \)
(l) The graph of a linear equation will always be a straight line \( T \)