Integrate the following:

1. \( \int x \sin 3x \, dx \)

2. \( \int_0^2 x^3 \sqrt{4 - 9x^2} \, dx \)

3. \( \int x \sqrt{x^2 + 1} \, dx \)

4. \( \int \frac{1}{x \sqrt{x + 1}} \, dx \) (Hint: make a “tricky” substitution to express the integrand as a rational function.)

5. Evaluate \( \int_1^\infty \frac{1}{2x^3} \, dx \). Shade the region represented by the integral.

6. Find the surface area of the solid obtained by rotating the curve \( y = \sqrt[3]{x} \) from \( x = 1 \) to \( x = 8 \) about the \( y \)-axis. Sketch the curve, the resulting solid of revolution, and a typical slice of surface area. Hint: understand your ability to choose which variable you work in.

7. The density of Lost Coast Brewery’s Alley Cat Amber Ale is 1070 kg/m\(^3\). A full keg of Alley Cat Amber Ale is in the shape of a cylinder whose height is 1 m, and whose radius is 0.5 m. (In the following calculations, we are discounting any force due to the carbonation of the beer.)
   a. Find the hydrostatic force on the bottom of the keg, if the keg is standing upright.
   b. Find the hydrostatic force on one end of the keg, if the keg is lying on its side.

8. Three point-masses are located on an \( x \)-axis. The first point-mass has a mass of 25 located at -10, the second point-mass has a mass of 4 located at -2, and the third point-mass has a mass of 12 located at 4. Sketch the masses in their places and find the moment \( M \) about the \( x \)-axis, and the center of mass, \( \bar{x} \).