Chapter 9: Center of Mass & Linear Momentum

Example Problems

\[ \vec{r}_{\text{com}} = \frac{1}{M} \sum m_i \vec{r}_i \quad \vec{P}_{\text{total}} = M \vec{v}_{\text{com}} \quad \Delta \vec{P} = \vec{J} = \int_{t_0}^{t_f} \vec{F}(t) dt \]

\[ \begin{align*}
\vec{p}_{10} + \vec{p}_{20} &= \vec{p}_{1f} + \vec{p}_{2f} \\
m_1 \vec{v}_{10} + m_2 \vec{v}_{20} &= m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \\
v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{10} + \frac{2m_2}{m_1 + m_2} v_{20} \\
v_{2f} &= \frac{2m_1}{m_1 + m_2} v_{10} + \frac{m_2 - m_1}{m_1 + m_2} v_{20} \quad \rightarrow \text{elastic}
\end{align*} \]

Example 9.1
a. Someone claims that when a skillful higher jumper clears the bar her or his center of mass actually goes under the bar. Is this possible?
b. It used to be common wisdom to build cars to be as rigid as possible to withstand collisions. Today, thought, areas are designed to have “crumpled zones” that collapse upon impact. What advantage does this have?
c. Can a sailboat be propelled by air blown at the sails from a fan attached to the boat? Explain your answer.
d. Consider the system consisting of the rifle plus the bullet. What is the speed of the center of mass of the system after the rifle is fired? Explain.
e. Why, when you release an inflated, untied balloon, does it fly across the room?
f. Is it easier to hit a home run from a pitched ball than from one tossed in the air by the batter? Explain.

Example 9.2
Romeo (77 kg) entertains Juliet (55 kg) by playing his guitar from the rear of their boat at rest in still water, 2.70 m away from Juliet who is in the front of the boat. After the serenade, Juliet carefully moves to the rear of the boat (away from shore) to plant a kiss on Romero’s cheek. How far does the 80 kg boat move toward the shore it is facing?

Example 9.3
Until he was in his seventies, Henri LaMothe excited audiences by belly-flopping from a height of 12 m into 30 cm of water. Assuming that he stops just as he reaches the bottom of the water and estimating his mass, find the magnitude of the impulse on him from the water.
Example 9.4
A 4.0 kg mess kit sliding on a frictionless surface explodes into two 2.0 kg parts, one moving at 3.0 m/s, due north, and the other at 5.0 m/s, 30° north of east. (a) Physically explain what direction the original particle comes from? (b) What is the original velocity of the mess kit?

Example 9.5
A completely inelastic collision occurs between two balls of wet putty that move directly toward each other along a vertical axis. Just before the collision, one ball, of mass 3.0 kg, is moving upward at 20 m/s and the other ball, of mass 2.0 kg, is moving downward at 12 m/s. How high do the combined two balls of putty rise above the collision point?

Example 9.6
Block 1 of mass $m_1$ slides along an x axis on a frictionless floor at speed 4.00 m/s. Then it undergoes a one-dimensional elastic collision with stationary block 2 of mass $m_2 = 2.00m_1$. Next, block 2 undergoes a one-dimensional elastic collision with stationary block 3 of mass $m_3 = 2.00m_2$. (a) Draw a momentum bar diagram for the situation and rank the final momentums of blocks 1, 2, and 3. (b) What is the speed of block 3 after the collisions? (c) Is the speed, kinetic energy, and momentum of block 3 greater than, less than, or the same as the initial values for block 1?
Example 9.7
Practice setting up these problems without actually solving them numerically.
A. Block-2 (1-kg) is at rest on a frictionless surface and touching the end of an unstretched spring of spring constant 200 N/m. The other end of the spring is fixed to a wall. Block-1 (2 kg), traveling at speed \( v_1 = 4.0 \text{ m/s} \), collides with block-2, and the two blocks stick together. When the blocks momentarily stop, by what distance is the spring compressed?
Answer: \( 0.33 \text{ m} \)

B. Block-1 of mass \( m_1 \) slides from rest along a frictionless ramp from \( h = 2.50 \text{ m} \) and then collides with stationary block-2, which mass \( m_2 = 2m_1 \). After the collision, block-2 slides into a region where the coefficient of kinetic friction \( \mu_k \) is 0.500 and comes to a stop in distance \( d \) within that region. What is the value of distance \( d \) if the collision is (a) elastic and (b) completely inelastic?
Answer: (a) \( 2.22 \text{ m} \); (b) \( 0.556 \text{ m} \)

C. A ball (0.060 kg) is shot with speed 22 m/s into the barrel of a spring gun (0.240 kg) initially at rest on a frictionless surface. The ball sticks in the barrel at the point of maximum compression of the spring. Assume that the increase in thermal energy due to friction between the ball and the barrel is negligible. (a) What is the speed of the spring gun after the ball stops in the barrel? (b) What fraction of the initial kinetic energy of the ball is stored in the spring?
Answer: (a) \( 4.4 \text{ m/s} \); (b) \( .80 \)

Solution to A
We think of this as having two parts: the first is the collision itself – where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet – and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount \( x_m \).

The collision is completely inelastic and conservation of momentum tells us
\[
    p_{10} + p_{20} = p_f \quad \Rightarrow \quad m_1 v_{10} = (m_1 + m_2) v_f \quad \Rightarrow \quad v_f = \frac{m_1 v_{10}}{m_1 + m_2} = 2.7 \text{ m/s} = v_f
\]

Applying conservation of energy to find the maximum compression,
\[
    K_{10} = U_s \quad \Rightarrow \quad \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} k_s x_{\text{max}}^2 \quad \Rightarrow \quad x_{\text{max}} = \sqrt{\frac{(m_1 + m_2) v_f^2}{k_s}}
\]

\[
    k_s = 200 \text{ N/m} \quad \Rightarrow \quad 0.33 \text{ m} = x_{\text{max}}
\]

Solution to B
a. No external forces and the collision is elastic, one then applies the elastic collision equations:
\[
    v_{2f} = \frac{2m_1}{m_1 + m_2} v_{10} + 0 \quad \Rightarrow \quad v_{10} = \frac{2m_1 \sqrt{2gh}}{m_2 + 2m_1} = 4.67 \text{ m/s}
\]

where we have used energy conservation to find the speed of block 1 at the bottom of the frictionless ramp (\( \sqrt{2gh} \)). After the collision, we apply energy conservation to find the distance \( d \) that block-2 travels on the "rough slide."
\[
\frac{K_{20}}{W_{NC}} \rightarrow \frac{1}{2} m_2 v_{20}^2 = \mu_k m g d \quad \text{solving for } d \quad d = \frac{\frac{1}{2} v_{20}^2}{\mu_k g} = 2.22 \text{ m} = d
\]

b. In a completely inelastic collision, we apply a different conservation of momentum equation:
\[
P_{10} + P_{20} = P_f \rightarrow m_1 v_{10} = (m_1 + m_2) v_f \quad \text{solving for } v_f \quad v_f = \frac{m_1}{m_1 + m_2} v_{10} \quad m_2 = 2m_1 \quad v_{10} = 4.67 \text{ m/s}
\]

Conservation of energy gives a distance of
\[
K_{20} = W_{NC} \rightarrow d = \frac{\frac{1}{2} v_{20}^2}{\mu_k g} = \frac{1}{\mu_k} = 0.56 \text{ m} = d
\]

Solution to C

a. During the collision between the ball and the spring gun, momentum is conserved. Since the collision is completely inelastic, we write
\[
P_{10} + P_{20} = P_f \rightarrow v_f = \frac{m_1}{m_1 + m_2} v_{10} \quad m_1 = 60 \text{ g} \quad m_2 = 240 \text{ g} \quad v_{10} = 22 \text{ m/s} \quad v_f = 4.4 \text{ m/s}
\]

b. According to conservation of energy, if \( W_{NC} \) is zero, then all of the initial kinetic energy went into moving the spring gun/ball system and into the spring: are
\[
\frac{K_{10}}{E_1} = \frac{K_f + U_s}{E_2} \rightarrow 1 = \frac{K_f}{K_{10}} + \frac{U_s}{K_{10}} \quad \frac{U_s}{K_{10}} = 1 - \frac{K_f}{K_{10}} = 1 - \frac{1}{2} \left( m_1 + m_2 \right) v_f^2 \quad \frac{0.80}{K_{10}} = \frac{U_s}{K_{10}}
\]

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is 80%.