\[ \mu_m = -2(2.00 \times 10^{-3} \text{ m})(2\pi)(806.2 \text{ Hz}) \]
\[ \times \sin\left(\frac{2\pi}{0.400 \text{ m}}(0.180 \text{ m})\right) \]
\[ = 6.26 \text{ m/s}. \quad \text{(Answer)} \]

To determine when the string element has this maximum speed, we could investigate Eq. 16-69. However, a little thought can save a lot of work. The element is undergoing SHM and must come to a momentary stop at its extreme upward position and extreme downward position. It has the greatest speed as it zips through the midpoint of its oscillation, just as a block does in a block–spring oscillator.

**Review & Summary**

**Transverse and Longitudinal Waves** Mechanical waves can exist only in material media and are governed by Newton's laws. Transverse mechanical waves, like those on a stretched string, are waves in which the particles of the medium oscillate perpendicular to the wave's direction of travel. Waves in which the particles of the medium oscillate parallel to the wave's direction of travel are longitudinal waves.

**Sinusoidal Waves** A sinusoidal wave moving in the positive direction of an x axis has the mathematical form

\[ y(x, t) = y_m \sin(kx - \omega t), \quad (16-2) \]

where \( y_m \) is the amplitude of the wave, \( k \) is the angular wave number, \( \omega \) is the angular frequency, and \( kx - \omega t \) is the phase. The wavelength \( \lambda \) is related to \( k \) by

\[ k = \frac{2\pi}{\lambda}. \quad (16-5) \]

The period \( T \) and frequency \( f \) of the wave are related to \( \omega \) by

\[ \frac{\omega}{2\pi} = f = \frac{1}{T}. \quad (16-9) \]

Finally, the wave speed \( v \) is related to these other parameters by

\[ v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f. \quad (16-13) \]

**Equation of a Traveling Wave** Any function of the form

\[ y(x, t) = h(kx \pm \omega t) \quad (16-17) \]

can represent a traveling wave with a wave speed given by Eq. 16-13 and a wave shape given by the mathematical form of \( h \). The plus sign denotes a wave traveling in the negative direction of the x axis, and the minus sign a wave traveling in the positive direction.

**Wave Speed on Stretched String** The speed of a wave on a stretched string is set by properties of the string. The speed on a string with tension \( T \) and linear density \( \mu \) is

\[ v = \sqrt{\frac{T}{\mu}}. \quad (16-26) \]

**Power** The average power of, or average rate at which energy is transmitted by, a sinusoidal wave on a stretched string is given by

\[ P_{\text{avg}} = \frac{1}{2} \mu v \omega y_m^2. \quad (16-33) \]

**Superposition of Waves** When two or more waves traverse the same medium, the displacement of any particle of the medium is the sum of the displacements that the individual waves would give it.

**Interference of Waves** Two sinusoidal waves on the same string exhibit interference, adding or canceling according to the principle of superposition. If the two are traveling in the same direction and have the same amplitude \( y_m \) and frequency (hence the same wavelength) but differ in phase by a phase constant \( \phi \), the result is a single wave with this same frequency:

\[ y'(x, t) = [2y_m \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi). \quad (16-51) \]

If \( \phi = 0 \), the waves are exactly in phase and their interference is fully constructive; if \( \phi = \pi \text{ rad} \), they are exactly out of phase and their interference is fully destructive.

**Phasors** A wave \( y(x, t) \) can be represented with a phasor. This is a vector that has a magnitude equal to the amplitude \( y_m \) of the wave and that rotates about an origin with an angular speed equal to the angular frequency \( \omega \) of the wave. The projection of the rotating phasor on a vertical axis gives the displacement \( y \) of a point along the wave’s travel.

**Standing Waves** The interference of two identical sinusoidal waves moving in opposite directions produces standing waves. For a string with fixed ends, the standing wave is given by

\[ y'(x, t) = [2y_m \sin kx] \cos \omega t. \quad (16-60) \]

Standing waves are characterized by fixed locations of zero displacement called nodes and fixed locations of maximum displacement called antinodes.

**Resonance** Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a resonant frequency, and the corresponding standing wave pattern is an oscillation mode. For a stretched string of length \( L \) with fixed ends, the resonant frequencies are

\[ f = \frac{v}{\lambda} = n \frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots. \quad (16-66) \]

The oscillation mode corresponding to \( n = 1 \) is called the fundamental mode or the first harmonic; the mode corresponding to \( n = 2 \) is the second harmonic; and so on.
1. The following four waves are sent along strings with the same linear densities (x is in meters and t is in seconds). Rank the waves according to (a) their wave speed and (b) the tension in the strings along which they travel, greatest first:

- (1) \( y_1 = (3 \text{ mm}) \sin(x - 3t) \)
- (2) \( y_2 = (6 \text{ mm}) \sin(2x - t) \)
- (3) \( y_3 = (1 \text{ mm}) \sin(4x - t) \)
- (4) \( y_4 = (2 \text{ mm}) \sin(x - 2t) \)

2. In Fig. 16-24, wave 1 consists of a rectangular peak of height 4 units and width 2 cm, and a rectangular valley of depth 2 units and width 2 mm. The wave travels rightward along an x axis. Choices 2, 3, and 4 are similar waves, with the same heights, depths, and widths, that will travel leftward along that axis and through wave 1. Right-going wave 1 and one of the left-going waves will interfere as they pass through each other. With which left-going wave will the interference give, for an instant, (a) the deepest valley, (b) a flat line, and (c) a flat peak 2 cm wide?

![Figure 16-24](image)

3. Figure 16-25a gives a snapshot of a wave traveling in the direction of positive x along a string under tension. Four string elements are indicated by the lettered points. For each of those elements, determine whether, at the instant of the snapshot, the element is moving upward or downward or is momentarily at rest. (Hint: Imagine the wave as it moves through the four string elements, as if you were watching a video of the wave as it traveled rightward.)

Figure 16-25b gives the displacement of a string element located at, say, \( x = 0 \) as a function of time. At the lettered times, is the element moving upward or downward or is it momentarily at rest?

![Figure 16-25](image)

4. Figure 16-26 shows three waves that are separately sent along a string that is stretched under a certain tension along an x axis. Rank the waves according to their (a) wavelengths, (b) speeds, and (c) angular frequencies, greatest first.

![Figure 16-26](image)

5. If you start with two sinusoidal waves of the same amplitude traveling in phase on a string and then somehow phase-shift one of them by 5.4 wavelengths, what type of interference will occur on the string?

6. The amplitudes and phase differences for four pairs of waves of equal wavelengths are (a) 2 mm, 6 mm, and \( \pi \text{ rad} \); (b) 3 mm, 5 mm, and \( \pi \text{ rad} \); (c) 7 mm, 9 mm, and \( \pi \text{ rad} \); (d) 2 mm, 2 mm, and 0 rad. Each pair travels in the same direction along the same string. Without written calculation, rank the four pairs according to the amplitude of their resultant wave, greatest first. (Hint: Construct phasor diagrams.)

7. A sinusoidal wave is sent along a cord under tension, transporting energy at the average rate of \( P_{\text{avg,1}} \). Two waves, identical to that first one, are then to be sent along the cord with a phase difference \( \phi \) of either 0.0, 0.2 wavelength, or 0.5 wavelength. (a) With only mental calculation, rank those choices of \( \phi \) according to the average rate at which the waves will transport energy, greatest first. (b) For the first choice of \( \phi \), what is the average rate in terms of \( P_{\text{avg,1}} \)?

8. If a standing wave on a string is given by \( y'(t) = (3 \text{ mm}) \sin(5x) \cos(4t) \), is there a node or an antinode of the oscillations of the string at \( x = 0 \)? (b) If the standing wave is given by \( y'(t) = (3 \text{ mm}) \sin(5x + \pi/2) \cos(4t) \), is there a node or an antinode at \( x = 0 \)?

9. Strings A and B have identical lengths and linear densities, but string B is under greater tension than string A. Figure 16-27 shows four situations, (a) through (d), in which standing wave patterns exist on the two strings. In which situations is there the possibility that strings A and B are oscillating at the same resonant frequency?

![Figure 16-27](image)

10. If you set up the seventh harmonic on a string, (a) how many nodes are present, and (b) is there a node, antinode, or some intermediate state at the midpoint? If you next set up the sixth harmonic, (c) is its resonant wavelength longer or shorter than that for the seventh harmonic, and (d) is the resonant frequency higher or lower?

11. Figure 16-28 shows phasor diagrams for three situations in which two waves travel along the same string. All six waves have the same amplitude. Rank the situations according to the amplitude of the net wave on the string, greatest first.

![Figure 16-28](image)
Module 16-1 Transverse Waves

1. If a wave \( y(x, t) = (6.0 \text{ mm}) \sin (kx + (600 \text{ rad/s})t + \phi) \) travels along a string, how much time does any given point on the string take to move between displacements \( y = +2.0 \text{ mm} \) and \( y = -2.0 \text{ mm} \)?

2. A human wave. During sporting events within large, densely packed stadiums, spectators will send a wave (or pulse) around the stadium (Fig. 16-29). As the wave reaches a group of spectators, they stand with a cheer and then sit. At any instant, the width \( w \) of the wave is the distance from the leading edge (people are just about to stand) to the trailing edge (people have just sat down). Suppose a human wave travels a distance of 853 seats around a stadium in 39 s, with spectators requiring about 1.8 s to respond to the wave’s passage by standing and then sitting. What are (a) the wave speed \( v \) (in seats per second) and (b) width \( w \) (in number of seats)?

3. A wave has an angular frequency of 110 rad/s and a wavelength of 1.80 m. Calculate (a) the angular wave number and (b) the speed of the wave.

4. A sand scorpion can detect the motion of a nearby beetle (its prey) by the waves the motion sends along the sand surface (Fig. 16-30). The waves are of two types: transverse waves traveling at \( v_1 = 50 \text{ m/s} \) and longitudinal waves traveling at \( v_2 = 150 \text{ m/s} \). If a sudden motion sends out such waves, a scorpion can tell the distance of the beetle from the difference \( \Delta t \) in the arrival times of the waves at its leg nearest the beetle. If \( \Delta t = 4.0 \text{ ms} \), what is the beetle’s distance?

5. A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero is 0.170 s. What are the (a) period and (b) frequency? (c) The wavelength is 1.40 m; what is the wave speed?

6. A sinusoidal wave travels along a string under tension. Figure 16-31 gives the slopes along the string at time \( t = 0 \). The scale of the \( x \) axis is set by \( x_s = 0.80 \text{ m} \). What is the amplitude of the wave?

7. A transverse sinusoidal wave is moving along a string in the positive direction of an \( x \) axis with a speed of 80 m/s. At \( t = 0 \), the string particle at \( x = 0 \) has a transverse displacement of 4.0 cm from its equilibrium position and is not moving. The maximum transverse speed of the string particle at \( x = 0 \) is \( 16 \text{ m/s} \). (a) What is the frequency of the wave? (b) What is the wavelength of the wave? If \( y(x, t) = y_m \sin (kx - \omega t + \phi) \) is the form of the wave equation, what are (c) \( y_m \), (d) \( k \), (e) \( \omega \), (f) \( \phi \), and (g) the correct choice of sign in front of \( \omega \)?

8. Figure 16-32 shows the transverse velocity \( u \) versus time \( t \) of the point on a string at \( x = 0 \), as a wave passes through it. The scale on the vertical axis is set by \( u_s = 4.0 \text{ m/s} \). The wave has the generic form \( y(x, t) = y_m \sin (kx - \omega t + \phi) \). What then is \( \phi \)? (Caution: A calculator does not always give the proper inverse trig function, so check your answer by substituting it and an assumed value of \( \omega \) into \( y(x, t) \) and then plotting the function.)

9. A sinusoidal wave moving along a string is shown twice in Fig. 16-33, as crest \( A \) travels in the positive direction of an \( x \) axis by distance \( d = 6.0 \text{ cm} \) in \( 4.0 \text{ ms} \). The tick marks along the axis are separated by \( 10 \text{ cm} \); height \( H = 6.00 \text{ mm} \). The equation for the wave is in the form \( y(x, t) = y_m \sin (kx - \omega t + \phi) \), so what are (a) \( y_m \), (b) \( k \), (c) \( \omega \), and (d) the correct choice of sign in front of \( \omega \)?

10. The equation of a transverse wave traveling along a very long string is \( y = 6.0 \sin(0.020 \pi x + 4.0\pi t) \), where \( x \) and \( y \) are expressed in centimeters and \( t \) is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the speed, (e) the direction of propagation of the wave, and (f) the maximum transverse speed of a particle in the string. (g) What is the transverse displacement at \( x = 3.5 \text{ cm} \) when \( t = 0.26 \text{ s} \)?

11. A sinusoidal transverse wave of wavelength 20 cm travels along a string in the positive direction of an \( x \) axis. The displacement \( y \) of the string particle at \( x = 0 \) is given in Fig. 16-34 as a function of time \( t \). The scale of the vertical axis is set by \( y_s = 4.0 \text{ cm} \). The wave equation is to be in the form \( y(x, t) = y_m \sin (kx + \omega t + \phi) \). (a) At \( t = 0 \), is a plot of \( y \) versus \( x \) in the shape of a positive sine function or a negative sine function? What are (b) \( y_m \), (c) \( k \), (d) \( \omega \), (e) \( \phi \), (f) the sign in front of \( \omega \), and (g) the speed of the wave? (h) What is the transverse velocity of the particle at \( x = 0 \) when \( t = 5.0 \text{ s} \)?

12. The function \( y(x, t) = (15.0 \text{ cm}) \cos(\pi x - 15\pi t) \), with \( x \) in meters and \( t \) in seconds, describes a wave on a taut string. What is
the transverse speed for a point on the string at an instant when that point has the displacement \( y = +12.0 \) cm?

**13 ILW** A sinusoidal wave of frequency 500 Hz has a speed of 350 m/s. (a) How far apart are two points that differ in phase by \( \pi /5 \) rad? (b) What is the phase difference between two displacements at a certain point at times 1.00 ms apart?

**Module 16-2 Wave Speed on a Stretched String**

**14** The equation of a transverse wave on a string is 
\[ y = (2.0 \text{ mm}) \sin[(20 \text{ m}^{-1}) x - (600 \text{ s}^{-1}) t]. \]

The tension in the string is 15 N. (a) What is the wave speed? (b) Find the linear density of this string in grams per meter.

**15 WWW** A stretched string has a mass per unit length of 5.00 g/cm and a tension of 10.0 N. A sinusoidal wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is traveling in the negative direction of an x axis. If the wave equation is of the form \( y(x, t) = y_m \sin(kx \pm \omega t) \), what are (a) \( y_m \), (b) \( k \), (c) \( \omega \), and (d) the correct choice of sign in front of \( \omega \)?

**16** The speed of a transverse wave on a string is 170 m/s when the string tension is 120 N. To what value must the tension be changed to raise the wave speed to 180 m/s?

**17** The linear density of a string is \( 1.6 \times 10^{-4} \) kg/m. A transverse wave on the string is described by the equation
\[ y = (0.021 \text{ m}) \sin[(2.0 \text{ m}^{-1}) x + (30 \text{ s}^{-1}) t]. \]

What are (a) the wave speed and (b) the tension in the string?

**18** The heaviest and lightest strings on a certain violin have linear densities of 3.0 and 0.29 g/m. What is the ratio of the diameter of the heaviest string to that of the lightest string, assuming that the strings are of the same material?

**19 SSM** What is the speed of a transverse wave in a rope of length 2.00 m and mass 60.0 g under a tension of 500 N?

**20** The tension in a wire clamped at both ends is doubled without appreciably changing the wire’s length between the clamps. What is the ratio of the new to the old wave speed for transverse waves traveling along this wire?

**21 ILW** A 100 g wire is held under a tension of 250 N with one end at \( x = 0 \) and the other at \( x = 10.0 \) m. At time \( t = 0 \), pulse 1 is sent along the wire from the end at \( x = 10.0 \) m. At time \( t = 30.0 \) ms, pulse 2 is sent along the wire from the end at \( x = 0 \). At what position \( x \) do the pulses begin to meet?

**22** A sinusoidal wave is traveling on a string with speed 40 cm/s. The displacement of the particles of the string at \( x = 10 \text{ cm} \) varies with time according to \( y = (5.0 \text{ cm}) \sin[1.0 - (4.0 \text{ s}^{-1}) t] \). The linear density of the string is 4.0 g/cm. What are (a) the frequency and (b) the wavelength of the wave? If the wave equation is of the form \( y(x, t) = y_m \sin(kx \pm \omega t) \), what are (c) \( y_m \), (d) \( k \), (e) \( \omega \), and (f) the correct choice of sign in front of \( \omega \)? (g) What is the tension in the string?

**23 SSM ILW** A sinusoidal transverse wave is traveling along a string in the negative direction of an x axis. Figure 16-35 shows a plot of the displacement as a function of position at time \( t = 0 \); the scale of the y axis is set by \( y_i = 4.0 \) cm. The string tension is 3.6 N, and its linear density is 25 g/m. Find the (a) amplitude, (b) wavelength, (c) wave speed, and (d) period of the wave. (e) Find the maximum transverse speed of a particle in the string. If the wave is of the form \( y(x, t) = y_m \sin(kx \pm \omega t + \phi) \), what are (f) \( k \), (g) \( \omega \), (h) \( \phi \), and (i) the correct choice of sign in front of \( \omega \)?

**24** In Fig. 16-36a, string 1 has a linear density of 3.00 g/m, and string 2 has a linear density of 5.00 g/m. They are under tension due to the hanging block of mass \( M = 500 \) g. Calculate the wave speed on (a) string 1 and (b) string 2. (Hint: When a string loops halfway around a pulley, it pulls on the pulley with a net force that is twice the tension in the string.) Next the block is divided into two blocks (with \( M_1 + M_2 = M \)) and the apparatus is rearranged as shown in Fig. 16-36b. Find (c) \( M_1 \) and (d) \( M_2 \) such that the wave speeds in the two strings are equal.

**25** A uniform rope of mass \( m \) and length \( L \) hangs from a ceiling. (a) Show that the speed of a transverse wave on the rope is a function of \( y \), the distance from the lower end, and is given by \( v = \sqrt{\frac{g}{y}} \). (b) Show that the time a transverse wave takes to travel the length of the rope is given by \( t = 2\sqrt{\frac{L}{g}} \).

**Module 16-3 Energy and Power of a Wave Traveling Along a String**

**26** A string along which waves can travel is 2.70 m long and has a mass of 260 g. The tension in the string is 36.0 N. What must be the frequency of traveling waves of amplitude 7.70 mm for the average power to be 85.0 W?

**27** A sinusoidal wave is sent along a string with a linear density of 2.0 g/m. As it travels, the kinetic energies of the mass elements along the string vary. Figure 16-37a gives the rate \( \frac{dK}{dt} \) at which kinetic energy passes through the string elements at a particular instant, plotted as a function of distance \( x \) along the string. Figure 16-37b is similar except that it gives the rate at which kinetic energy passes through a particular mass element (at a particular location), plotted as a function of time \( t \). For both figures, the scale on the vertical (rate) axis is set by \( R_y = 10 \) W. What is the amplitude of the wave?
Module 16-4  The Wave Equation

28 Use the wave equation to find the speed of a wave given by
\[ y(x, t) = (3.00 \text{ mm}) \sin[(4.00 \text{ m}^{-1})x - (7.00 \text{ s}^{-1})t]. \]

29 Use the wave equation to find the speed of a wave given by
\[ y(x, t) = (2.00 \text{ mm})[(20 \text{ m}^{-1})x - (4.00 \text{ s}^{-1})t]^2. \]

30 Use the wave equation to find the speed of a wave given in terms of the general function \( h(x, t) \):
\[ y(x, t) = (4.00 \text{ mm}) h[(30 \text{ m}^{-1})x + (6.0 \text{ s}^{-1})t]. \]

Module 16-5  Interference of Waves

31 SSM Two identical traveling waves, moving in the same direction, are out of phase by \( \pi/2 \) rad. What is the amplitude of the resultant wave in terms of the common amplitude \( y_m \) of the two combining waves?

32 What phase difference between two identical traveling waves, moving in the same direction along a stretched string, results in the combined wave having an amplitude 1.50 times that of the common amplitude of the two combining waves? Express your answer in (a) degrees, (b) radians, and (c) wavelengths.

33 Two sinusoidal waves with the same amplitude of 9.00 mm and the same wavelength travel together along a string that is stretched along an \( x \) axis. Their resultant wave is shown twice in Fig. 16-38, as valley B travels in the negative direction of the \( x \) axis by distance \( d = 56.0 \text{ cm} \) in 8.0 ms. The tick marks along the axis are separated by 10 cm, and height \( H \) is 8.0 mm. Let the equation for one wave be of the form \( y(x, t) = y_m \sin(kx \pm \omega t + \phi_1) \), where \( \phi_1 = 0 \) and you must choose the correct sign in front of \( \omega \). For the equation for the other wave, what are (a) \( y_m \), (b) \( k \), (c) \( \omega \), (d) \( \phi_2 \), and (e) the sign in front of \( \omega t \)?

34 A sinusoidal wave of angular frequency 1200 rad/s and amplitude 3.00 mm is sent along a cord with linear density 2.00 g/m and tension 1200 N. (a) What is the average rate at which energy is transported by the wave to the opposite end of the cord? (b) If, simultaneously, an identical wave travels along an adjacent, identical cord, what is the total average rate at which energy is transported to the opposite ends of the two cords by the waves? If, instead, those two waves are sent along the same cord simultaneously, what is the total average rate at which they transport energy when their phase difference is (c) 0, (d) 0.4 \( \pi \) rad, and (e) \( \pi \) rad?

Module 16-6  Phasors

35 SSM Two sinusoidal waves of the same frequency travel in the same direction along a string. If \( y_{m1} = 3.0 \text{ cm} \), \( y_{m2} = 4.0 \text{ cm} \), \( \phi_1 = 0 \), and \( \phi_2 = \pi/2 \) rad, what is the amplitude of the resultant wave?

36 Four waves are to be sent along the same string, in the same direction:
\[ y_1(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t) \]
\[ y_2(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t + 0.7\pi) \]
\[ y_3(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t + \pi) \]
\[ y_4(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t + 1.7\pi). \]

What is the amplitude of the resultant wave?

37 These two waves travel along the same string:
\[ y_1(x, t) = (4.60 \text{ mm}) \sin(2\pi x - 400\pi t) \]
\[ y_2(x, t) = (5.60 \text{ mm}) \sin(2\pi x - 400\pi t + 0.80\pi \text{ rad}). \]

What are (a) the amplitude and (b) the phase angle (relative to wave 1) of the resultant wave? (c) If a third wave of amplitude 5.00 mm is also to be sent along the string in the same direction as the first two waves, what should be its phase angle in order to maximize the amplitude of the new resultant wave?

38 Two sinusoidal waves of the same frequency are to be sent in the same direction along a taut string. One wave has an amplitude of 5.0 mm, the other 8.0 mm. (a) What phase difference \( \phi_1 \) between the two waves results in the smallest amplitude of the resultant wave? (b) What is that smallest amplitude? (c) What phase difference \( \phi_2 \) results in the largest amplitude of the resultant wave? (d) What is that largest amplitude? (e) What is the resultant amplitude if the phase angle is \( \phi_1 - \phi_2)/2? \)

39 Two sinusoidal waves of the same period, with amplitudes of 5.0 and 7.0 mm, travel in the same direction along a stretched string; they produce a resultant wave with an amplitude of 9.0 mm. The phase constant of the 5.0 mm wave is 0. What is the phase constant of the 7.0 mm wave?

Module 16-7  Standing Waves and Resonance

40 Two sinusoidal waves with identical wavelengths and amplitudes travel in opposite directions along a string with a speed of 10 cm/s. If the time interval between instants when the string is flat is 0.50 s, what is the wavelength of the waves?

41 SSM A string fixed at both ends is 8.40 m long and has a mass of 0.120 kg. It is subjected to a tension of 96.0 N and set oscillating. (a) What is the speed of the waves on the string? (b) What is the longest possible wavelength for a standing wave? (c) Give the frequency of that wave.

42 A string under tension \( \tau \) oscillates in the third harmonic at frequency \( f_3 \), and the waves on the string have wavelength \( \lambda_3 \). If the tension is increased to \( \tau' = 4\tau \), and the string is again made to oscillate in the third harmonic, what then are (a) the frequency of oscillation in terms of \( f_3 \) and (b) the wavelength of the waves in terms of \( \lambda_3 \)?

43 SSM WWW What are (a) the lowest frequency, (b) the second lowest frequency, and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long, has a mass of 100 g, and is stretched under a tension of 250 N?

44 A 125 cm length of string has mass 2.00 g and tension 7.00 N. (a) What is the wave speed for this string? (b) What is the lowest resonant frequency of this string?

45 SSM ILW A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 and 315 Hz, with no intermediate resonant frequencies. What are (a) the lowest resonant frequency and (b) the wave speed?

46 String \( A \) is stretched between two clamps separated by distance \( L \). String \( B \), with the same linear density and under the same tension as string \( A \), is stretched between two clamps separated by distance \( 4L \). Consider the first eight harmonics of string \( B \). For which of these eight harmonics of \( B \) (if any) does the frequency match the frequency of (a) \( A \)'s first harmonic, (b) \( A \)'s second harmonic, and (c) \( A \)'s third harmonic?

47 One of the harmonic frequencies for a particular string under tension is 325 Hz. The next higher harmonic frequency is 390 Hz.
What harmonic frequency is next higher after the harmonic frequency 195 Hz?

48 If a transmission line in a cold climate collects ice, the increased diameter tends to cause vortex formation in a passing wind. The air pressure variations in the vortexes tend to cause the line to oscillate (gallop), especially if the frequency of the variations matches a resonant frequency of the line. In long lines, the resonant frequencies are so close that almost any wind speed can set up a resonant mode vigorous enough to pull down support towers or cause the line to short out with an adjacent line. If a transmission line has a length of 347 m, a linear density of 3.35 kg/m, and a tension of 65.2 MN, what are (a) the frequency of the fundamental mode and (b) the frequency difference between successive modes?

49 A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance $D = 90.0$ cm apart. The string is oscillating in the standing wave pattern shown in Fig. 16-39. Calculate the (a) speed, (b) wavelength, and (c) frequency of the traveling waves whose superposition gives this standing wave.

50 For a particular transverse standing wave on a long string, one of the antinodes is at $x = 0$ and an adjacent node is at $x = 0.10$ m. The displacement $y(t)$ of the string particle at $x = 0$ is shown in Fig. 16-40, where the scale of the $y$ axis is set by $y_0 = 4.0$ cm. When $t = 0.50$ s, what is the displacement of the string particle at (a) $x = 0.20$ m and (b) $x = 0.30$ m? What is the transverse velocity of the string particle at $x = 0.20$ m at (c) $t = 0.50$ s and (d) $t = 1.0$ s? (e) Sketch the standing wave at $t = 0.50$ s for the range $x = 0$ to $x = 0.40$ m.

51 Two waves are generated on a string of length 3.0 m to produce a three-loop standing wave with an amplitude of 1.0 cm. The wave speed is 100 m/s. Let the equation for one of the waves be of the form $y(x, t) = y_m \sin(kx + \omega t)$. In the equation for the other wave, what are (a) $y_m$, (b) $k$, (c) $\omega$, and (d) the sign in front of $\omega$?

52 A rope, under a tension of 200 N and fixed at both ends, oscillates in a second-harmonic standing wave pattern. The displacement of the rope is given by

$$y = (0.10 \text{ m})(\sin \frac{\pi x}{2}) \sin 12 \pi t,$$

where $x = 0$ at one end of the rope, $x$ is in meters, and $t$ is in seconds. What are (a) the length of the rope, (b) the speed of the waves on the rope, and (c) the mass of the rope? (d) If the rope oscillates in a third-harmonic standing wave pattern, what will be the period of oscillation?

53 A string oscillates according to the equation

$$y' = (0.50 \text{ cm}) \sin \left( \frac{\pi}{3} \text{ cm}^{-1} \right) x \cos(40 \pi s^{-1}) t.$$

What are the (a) amplitude and (b) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation? (c) What is the distance between nodes? (d) What is the transverse speed of a particle of the string at the position $x = 1.5$ cm when $t = \frac{1}{8}$ s?

54 Two sinusoidal waves with the same amplitude and wavelength travel through each other along a string that is stretched along an $x$ axis. Their resultant wave is shown twice in Fig. 16-41, as the antinode $A$ travels from an extreme upward displacement to an extreme downward displacement in 6.0 ms. The tick marks along the axis are separated by 10 cm; height $H$ is 1.80 cm. Let the equation for one of the two waves be of the form $y(x, t) = y_m \sin(kx + \omega t)$. In the equation for the other wave, what are (a) $y_m$, (b) $k$, (c) $\omega$, and (d) the sign in front of $\omega$?

55 The following two waves are sent in opposite directions on a horizontal string so as to create a standing wave in a vertical plane:

$$y_1(x, t) = (6.00 \text{ mm}) \sin(4.00 \pi x - 400\pi t), \quad y_2(x, t) = (6.00 \text{ mm}) \sin(4.00 \pi x + 400\pi t),$$

with $x$ in meters and $t$ in seconds. An antinode is located at point $A$. In the time interval that point takes to move from maximum upward displacement to maximum downward displacement, how far does each wave move along the string?

56 A standing wave pattern on a string is described by

$$y(x, t) = 0.040 \,(\sin 5\pi x)(\cos 40\pi t),$$

where $x$ and $y$ are in meters and $t$ is in seconds. For $x \geq 0$, what is the location of the node with the (a) smallest, (b) second smallest, and (c) third smallest value of $x$? (d) What is the period of the oscillatory motion of any (nonnode) point? What are the (e) speed and (f) amplitude of the two traveling waves that interfere to produce this wave? For $t \geq 0$, what are the (g) first, (h) second, and (i) third time that all points on the string have zero transverse velocity?

57 A generator at one end of a very long string creates a wave given by

$$y = (6.00 \text{ cm}) \cos \frac{\pi}{2} [(2.00 \text{ m}^2) x + (8.00 \text{ s}^{-2}) t],$$

and a generator at the other end creates the wave

$$y = (6.00 \text{ cm}) \cos \frac{\pi}{2} [(2.00 \text{ m}^2) x - (8.00 \text{ s}^{-2}) t].$$

Calculate the (a) frequency, (b) wavelength, and (c) speed of each wave. For $x \geq 0$, what is the location of the node having the (d) smallest, (e) second smallest, and (f) third smallest value of $x$? For $x \geq 0$, what is the location of the antinode having the (g) smallest, (h) second smallest, and (i) third smallest value of $x$?

58 In Fig. 16-42, a string tied to a sinusoidal oscillator at $P$ and running over a support at $Q$, is stretched by a block of mass $m$ placed on a horizontal string so as to create a standing wave in a vertical plane.

$$y = (0.10 \text{ m})(\sin \frac{\pi x}{2}) \sin 12 \pi t,$$

where $x = 0$ at one end of the rope, $x$ is in meters, and $t$ is in seconds. What are (a) the length of the rope, (b) the speed of the waves on the rope, and (c) the mass of the rope? (d) If the rope oscillates in a third-harmonic standing wave pattern, what will be the period of oscillation?

59 A string oscillates according to the equation

$$y' = (0.50 \text{ cm}) \sin \left( \frac{\pi}{3} \text{ cm}^{-1} \right) x \cos(40 \pi s^{-1}) t.$$

What are the (a) amplitude and (b) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation? (c) What is the distance between nodes? (d) What is the transverse speed of a particle of the string at the position $x = 1.5$ cm when $t = \frac{1}{8}$ s?
frequency $f = 120$ Hz. The amplitude of the motion at $P$ is small enough for that point to be considered a node. A node also exists at $Q$. (a) What mass $m$ allows the oscillator to set up the fourth harmonic on the string? (b) What standing wave mode, if any, can be set up if $m = 1.00$ kg?

65 The equation of a transverse wave traveling along a string is

$$y = (2.0 \text{ mm}) \sin[(20 \text{ m}^{-1})x - (600 \text{ s}^{-1})t].$$

Find the (a) amplitude, (b) frequency, (c) velocity (including sign), and (d) wavelength of the wave. (e) Find the maximum transverse speed of a particle on the string.

66 Figure 16-44 shows the displacement $y$ versus time $t$ of the point on a string at $x = 0$, as a wave passes through that point. The scale of the $y$ axis is set by $y_s = 6.00 \text{ mm}$. The wave is given by $y(x, t) = y_m \sin(kx - \omega t + \phi)$. What is $\phi$? (Caution: A calculator does not always give the proper inverse trig function, so check your answer by substituting it and an assumed value of $\omega$ into $y(x, t)$ and then plotting the function.)

67 Two sinusoidal waves, identical except for phase, travel in the same direction along a string, producing the net wave $y(x, t) = (3.0 \text{ mm}) \sin(20\pi x - 4.0\pi t + 0.820 \text{ rad})$, with $x$ in meters and $t$ in seconds. What are (a) the wavelength $\lambda$ of the two waves, (b) the phase difference between them, and (c) their amplitude $y_m$?

68 A single pulse, given by $h(x - 5.0t)$, is shown in Fig. 16-45 for $t = 0$. The scale of the vertical axis is set by $h_i = 2$. Here $x$ is in centimeters and $t$ is in seconds. What are the (a) speed and (b) direction of travel of the pulse? (c) Plot $h(x - 5t)$ as a function of $x$ for $t = 2.0 \text{ s}$. (d) Plot $h(x - 5t)$ as a function of $t$ for $x = 10 \text{ cm}$.

69 SSM Three sinusoidal waves of the same frequency travel along a string in the positive direction of an $x$ axis. Their amplitudes are $y_1, y_2/2$, and $y_3/3$, and their phase constants are $0, \pi/2$, and $\pi$, respectively. What are the (a) amplitude and (b) phase constant of the resultant wave? (c) Plot the wave form of the resultant wave at $t = 0$, and discuss its behavior as $t$ increases.

70 Figure 16-46 shows transverse acceleration $a_x$ versus time $t$ of the point on a string at $x = 0$, as a wave in the form of $y(x, t) = y_m \sin(kx - \omega t + \phi)$ passes through that point. The scale of the vertical axis is set by $a_i = 400 \text{ m/s}^2$. What is $\phi$? (Caution: A calculator does not always give the proper inverse trig function, so check your answer by substituting it and an assumed value of $\omega$ into $y(x, t)$ and then plotting the function.)
Two sinusoidal 120 Hz waves, of the same frequency and amplitude, are to be sent in the positive direction of an x axis that is directed along a cord under tension. The waves can be sent in phase, or they can be phase-shifted. Figure 16-47 shows the amplitude y' of the resulting wave versus the distance of the shift (how far one wave is shifted from the other wave). The scale of the vertical axis is set by y' = 6.0 mm. If the equations for the two waves are of the form \( y(x, t) = y_m \sin(kx \pm \omega t) \), what are (a) \( y_m \), (b) \( k \), (c) \( \omega \), and (d) the correct choice of sign in front of \( \omega \)?

Energy is transmitted at rate \( P_1 \) by a wave of frequency \( f_1 \) on a string under tension \( \tau_1 \). What is the new energy transmission rate \( P_2 \) in terms of \( P_1 \) (a) if the tension is increased to \( \tau_2 = 4 \tau_1 \) and (b) if, instead, the frequency is decreased to \( f_2 = f_1/2 \)?

What is the fastest transverse wave that can be sent along a steel wire? For safety reasons, the maximum tensile stress to which steel wires should be subjected is \( 7.00 \times 10^8 \) N/m\(^2\). The density of steel is 7800 kg/m\(^3\). (b) Does your answer depend on the diameter of the wire?

A standing wave results from the sum of two transverse traveling waves given by

\[
y_1 = 0.050 \cos(\pi x - 4\pi t) \]

and

\[
y_2 = 0.050 \cos(\pi x + 4\pi t),
\]

where \( x \), \( y_1 \), and \( y_2 \) are in meters and \( t \) is in seconds. (a) What is the smallest positive value of \( x \) that corresponds to a node? (b) What is the value of the (b) first, (c) second, and (d) third time the particle at \( x = 0 \) has zero velocity?

The type of rubber band used inside some baseballs and golf balls obeys Hooke’s law over a wide range of elongation of the band. This type of material has an unstretched length \( \ell \) and a mass \( m \). When a force \( F \) is applied, the band stretches an additional length \( \Delta \ell \). (a) What is the speed (in terms of \( m \), \( \Delta \ell \), and the spring constant \( k \)) of transverse waves on this stretched rubber band? (b) Using your answer to (a), show that the time required for a transverse pulse to travel the length of the rubber band is proportional to \( 1/\sqrt{\Delta \ell} \) if \( \Delta \ell \ll \ell \) and is constant if \( \Delta \ell \gg \ell \).

The speed of electromagnetic waves (which include visible light, radio, and x rays) in vacuum is \( 3.0 \times 10^8 \) m/s. (a) Wavelengths of visible light waves range from about 400 nm in the violet to about 700 nm in the red. What is the range of frequencies of these waves? (b) The rate of frequencies for shortwave radio (for example, FM radio and VHF television) is 1.5 to 30 MHz. What is the corresponding wavelength range? (c) X-ray wavelengths range from about 5.0 nm to about \( 1.0 \times 10^{-2} \) nm. What is the frequency range for x rays?
pulse, moving at a slower speed \( v_p \), is due to the denting. As the projectile increases the dent's depth, the dent increases in radius, causing the material in the fibers to move in the same direction as the projectile (perpendicular to the transverse pulse's direction of travel). The rest of the projectile's energy goes into this motion. All the energy that does not eventually go into permanently deforming the fibers ends up as thermal energy.

Figure 16-48b is a graph of speed \( v \) versus time \( t \) for a bullet of mass 10.2 g fired from a .38 Special revolver directly into body armor. The scales of the vertical and horizontal axes are set by \( v_s = 300 \text{ m/s} \) and \( t_s = 40.0 \mu\text{s} \). Take \( v_t = 2000 \text{ m/s} \), and assume that the half-angle \( \theta \) of the conical dent is 60°. At the end of the collision, what are the radii of (a) the thinned region and (b) the dent (assuming that the person wearing the armor remains stationary)?

92 Two waves, \( y_1 = (2.50 \text{ mm}) \sin[(25.1 \text{ rad/m})x - (440 \text{ rad/s})t] \) and \( y_2 = (1.50 \text{ mm}) \sin[(25.1 \text{ rad/m})x + (440 \text{ rad/s})t] \), travel along a stretched string. (a) Plot the resultant wave as a function of \( t \) for \( x = 0, \lambda/8, \lambda/4, \lambda/8, \) and \( \lambda/2 \), where \( \lambda \) is the wavelength. The graphs should extend from \( t = 0 \) to a little over one period. (b) The resultant wave is the superposition of a standing wave and a traveling wave. In which direction does the traveling wave move? (c) How can you change the original waves so that the resultant wave is the superposition of standing and traveling waves with the same amplitudes as before but with the traveling wave moving in the opposite direction? Next, use your graphs to find the place at which the oscillation amplitude is (d) maximum and (e) minimum. (f) How is the maximum amplitude related to the amplitudes of the original two waves? (g) How is the minimum amplitude related to the amplitudes of the original two waves?

93 A traveling wave on a string is described by

\[
y = 2.0 \sin \left[ 2\pi \left( \frac{t}{0.40} + \frac{x}{80} \right) \right].
\]

where \( x \) and \( y \) are in centimeters and \( t \) is in seconds. (a) For \( t = 0 \), plot \( y \) as a function of \( x \) for \( 0 \leq x \leq 160 \text{ cm} \). (b) Repeat (a) for \( t = 0.05 \text{ s} \) and \( t = 0.10 \text{ s} \). From your graphs, determine (c) the wave speed and (d) the direction in which the wave is traveling.

94 In Fig. 16-50, a circular loop of string is set spinning about the center point in a place with negligible gravity. The radius is 4.00 cm and the tangential speed of a string segment is 5.00 cm/s. The string is plucked. At what speed do transverse waves move along the string? (Hint: Apply Newton’s second law to a small, but finite, section of the string.)

95 A continuous traveling wave with amplitude \( A \) is incident on a boundary. The continuous reflection, with a smaller amplitude \( B \), travels back through the incoming wave. The resulting interference pattern is displayed in Fig. 16-51. The standing wave ratio is defined to be

\[
\text{SWR} = \frac{A + B}{A - B}.
\]

The reflection coefficient \( R \) is the ratio of the power of the reflected wave to the power of the incoming wave and is thus proportional to the ratio \( (B/A)^2 \). What is the SWR for (a) total reflection and (b) no reflection? (c) For SWR = 1.50, what is \( R \) expressed as a percentage?

96 Consider a loop in the standing wave created by two waves (amplitude 5.00 mm and frequency 120 Hz) traveling in opposite directions along a string with length 2.25 m and mass 125 g and under tension 40 N. At what rate does energy enter the loop from (a) each side and (b) both sides? (c) What is the maximum kinetic energy of the string in the loop during its oscillation?