Sample Problem 2.06  Graphical integration a versus t, whiplash injury

“Whiplash injury” commonly occurs in a rear-end collision where a front car is hit from behind by a second car. In the 1970s, researchers concluded that the injury was due to the occupant’s head being whipped back over the top of the seat as the car was slammed forward. As a result of this finding, head restraints were built into cars, yet neck injuries in rear-end collisions continued to occur.

In a recent test to study neck injury in rear-end collisions, a volunteer was strapped to a seat that was then moved abruptly to simulate a collision by a rear car moving at 10.5 km/h. Figure 2-15a gives the accelerations of the volunteer’s torso and head during the collision, which began at time \( t = 0 \). The torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the volunteer. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate?

**KEY IDEA**

We can calculate the torso speed at any time by finding an area on the torso \( a(t) \) graph.

**Calculations:** We know that the initial torso speed is \( v_0 = 0 \) at time \( t_0 = 0 \), at the start of the “collision.” We want the torso speed \( v_1 \) at time \( t_1 = 110 \) ms, which is when the head begins to accelerate.

Combining Eqs. 2-27 and 2-28, we can write

\[
v_1 - v_0 = \left( \text{area between acceleration curve and time axis, from } t_1 \text{ to } t_1 \right).
\]

For convenience, let us separate the area into three regions (Fig. 2-15b). From 0 to 40 ms, region \( A \) has no area:

\[\text{area}_A = 0.\]

From 40 ms to 100 ms, region \( B \) has the shape of a triangle, with area

\[\text{area}_B = \frac{1}{2}(0.060 \text{ s})(50 \text{ m/s}^2) = 1.5 \text{ m/s}.\]

From 100 ms to 110 ms, region \( C \) has the shape of a rectangle, with area

\[\text{area}_C = (0.010 \text{ s})(50 \text{ m/s}^2) = 0.50 \text{ m/s}.\]

Substituting these values and \( v_0 = 0 \) into Eq. 2-31 gives us

\[v_1 - 0 = 0 + 1.5 \text{ m/s} + 0.50 \text{ m/s},\]

or

\[v_1 = 2.0 \text{ m/s} = 7.2 \text{ km/h}.\]

**Comments:** When the head is just starting to move forward, the torso already has a speed of 7.2 km/h. Researchers argue that it is this difference in speeds during the early stage of a rear-end collision that injures the neck. The backward whipping of the head happens later and could, especially if there is no head restraint, increase the injury.

Additional examples, video, and practice available at WileyPLUS.

**Review & Summary**

**Position**  The *position* \( x \) of a particle on an \( x \) axis locates the particle with respect to the *origin*, or zero point, of the axis. The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The *positive direction* on an axis is the direction of increasing positive numbers; the opposite direction is the *negative direction* on the axis.

**Displacement**  The *displacement* \( \Delta x \) of a particle is the change in its position:

\[\Delta x = x_2 - x_1.\]

Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the \( x \) axis and negative if the particle has moved in the negative direction.

**Average Velocity**  When a particle has moved from position \( x_1 \) to position \( x_2 \) during a time interval \( \Delta t = t_2 - t_1 \), its *average velocity* during that interval is

\[v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.\]

The algebraic sign of \( v_{\text{avg}} \) indicates the direction of motion (\( v_{\text{avg}} \) is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.

On a graph of \( x \) versus \( t \), the average velocity for a time interval \( \Delta t \) is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.
**Average Speed**  The average speed $s_{\text{avg}}$ of a particle during a time interval $\Delta t$ depends on the total distance the particle moves in that time interval:

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t} \quad \text{(2-3)}$$

**Instantaneous Velocity**  The instantaneous velocity (or simply velocity) $v$ of a moving particle is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \text{(2-4)}$$

where $\Delta x$ and $\Delta t$ are defined by Eq. 2-2. The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of $x$ versus $t$. **Speed** is the magnitude of instantaneous velocity.

**Average Acceleration**  Average acceleration is the ratio of a change in velocity $\Delta v$ to the time interval $\Delta t$ in which the change occurs:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} \quad \text{(2-7)}$$

The algebraic sign indicates the direction of $a_{\text{avg}}$.

**Instantaneous Acceleration**  Instantaneous acceleration (or simply acceleration) $a$ is the first time derivative of velocity $v(t)$ and the second time derivative of position $x(t)$:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad \text{(2-8, 2-9)}$$

On a graph of $v$ versus $t$, the acceleration $a$ at any time $t$ is the slope of the curve at the point that represents $t$.

**Constant Acceleration**  The five equations in Table 2-1 describe the motion of a particle with constant acceleration:

$$v = v_0 + at \quad \text{(2-11)}$$

$$x = x_0 + \frac{1}{2}at^2 \quad \text{(2-15)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \text{(2-16)}$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t \quad \text{(2-17)}$$

$$x - x_0 = vt - \frac{1}{2}at^2 \quad \text{(2-18)}$$

These are not valid when the acceleration is not constant.

**Free-Fall Acceleration**  An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth’s surface. The constant acceleration equations describe this motion, but we make two changes in notation: (1) we refer the motion to the vertical $y$ axis with $+y$ vertically up; (2) we replace $a$ with $-g$, where $g$ is the magnitude of the free-fall acceleration. Near Earth’s surface, $g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$.

### Questions

1. Figure 2-16 gives the velocity of a particle moving on an $x$ axis. What are (a) the initial and (b) the final directions of travel?  (c) Does the particle stop momentarily?  (d) Is the acceleration positive or negative?  (e) Is it constant or varying?

2. Figure 2-17 gives the acceleration $a(t)$ of a Chihuahua as it chases a German shepherd along an axis. In which of the time periods indicated does the Chihuahua move at constant speed?

3. Figure 2-18 shows four paths along which objects move from a starting point to a final point, all in the same time interval. The paths pass over a grid of equally spaced straight lines. Rank the paths according to (a) the average velocity of the objects and (b) the average speed of the objects, greatest first.

4. Figure 2-19 is a graph of a particle’s position along an $x$ axis versus time. (a) At time $t = 0$, what is the sign of the particle’s position?  Is the particle’s velocity positive, negative, or 0 at (b) $t = 1 \text{ s}$, (c) $t = 2 \text{ s}$, and (d) $t = 3 \text{ s}$?  (e) How many times does the particle go through the point $x = 0$?

5. Figure 2-20 gives the velocity of a particle moving along an axis. Point 1 is at the highest point on the curve; point 4 is at the lowest point; and points 2 and 6 are at the same height. What is the direction of travel at (a) time $t = 0$ and (b) point 4?  (c) At which of the six numbered points does the particle reverse its direction of travel?  (d) Rank the six points according to the magnitude of the acceleration, greatest first.

6. At $t = 0$, a particle moving along an $x$ axis is at position $x_0 = -20 \text{ m}$. The signs of the particle’s initial velocity $v_0$ (at time $t_0$) and constant acceleration $a$ are, respectively, for four situations: (1) $+, +$; (2) $-, -$; (3) $-, +$; (4) $-, -$. In which situations will the particle (a) stop momentarily, (b) pass through the origin, and (c) never pass through the origin?

7. Hanging over the railing of a bridge, you drop an egg (no initial velocity) as you throw a second egg downward. Which curves in Fig. 2-21
give the velocity \( v(t) \) for (a) the dropped egg and (b) the thrown egg? (Curves A and B are parallel; so are C, D, and E; so are F and G.)

8 The following equations give the velocity \( v(t) \) of a particle in four situations: (a) \( v = 3 \); (b) \( v = 4t^2 + 2t - 6 \); (c) \( v = 3t - 4 \); (d) \( v = 5t^2 - 3 \). To which of these situations do the equations of Table 2-1 apply?

9 In Fig. 2-22, a cream tangerine is thrown directly upward past three evenly spaced windows of equal heights. Rank the windows according to (a) the average speed of the cream tangerine while passing them, (b) the time the cream tangerine takes to pass them, (c) the magnitude of the acceleration of the cream tangerine while passing them, and (d) the change \( \Delta v \) in the speed of the cream tangerine during the passage, greatest first.

10 Suppose that a passenger intent on lunch during his first ride in a hot-air balloon accidently drops an apple over the side during the balloon’s liftoff. At the moment of the apple’s release, the balloon is accelerating upward with a magnitude of 4.0 m/s\(^2\) and has an upward velocity of magnitude 2 m/s. What are the (a) magnitude and (b) direction of the acceleration of the apple just after it is released? (c) Just then, is the apple moving upward or downward, or is it stationary? (d) What is the magnitude of its velocity just then? (e) In the next few moments, does the speed of the apple increase, decrease, or remain constant?

11 Figure 2-23 shows that a particle moving along an \( x \) axis undergoes three periods of acceleration. Without written computation, rank the acceleration periods according to the increases they produce in the particle’s velocity, greatest first.

**Problems**

**Module 2-1 Position, Displacement, and Average Velocity**

1 While driving a car at 90 km/h, how far do you move while your eyes shut for 0.50 s during a hard sneeze?

2 Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph \( x \) versus \( t \) for both cases and indicate how the average velocity is found on the graph.

3 An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during the full 80 km trip? (Assume that it moves in the positive \( x \) direction.) (b) What is the average speed? (c) Graph \( x \) versus \( t \) and indicate how the average velocity is found on the graph.

4 A car moves uphill at 40 km/h and then back downhill at 60 km/h. What is the average speed for the round trip?

5 The position of an object moving along an \( x \) axis is given by \( x = 3t - 4t^2 + t^3 \), where \( x \) is in meters and \( t \) in seconds. Find the position of the object at the following values of \( t \): (a) 1 s, (b) 2 s, (c) 3 s, and (d) 4 s. (e) What is the object’s displacement between \( t = 0 \) and \( t = 4 \) s? (f) What is its average velocity for the time interval from \( t = 2 \) s to \( t = 4 \) s? (g) Graph \( x \) versus \( t \) for \( 0 \leq t \leq 4 \) s and indicate how the answer for (f) can be found on the graph.

6 The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s, at which he commented, “Cogito ergo zoom!” (I think, therefore I go fast!). In 2001, Sam Whittingham beat Huber’s record by 19.0 km/h. What was Whittingham’s time through the 200 m?

7 Two trains, each having a speed of 30 km/h, are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the (crazy) bird flies directly back to the first train, and so forth. What is the total distance the bird travels before the trains collide?

8 Panic escape. Figure 2-24 shows a general situation in which a stream of people attempt to escape through an exit door that turns out to be locked. The people move toward the door at speed \( v_i = 3.50 \) m/s, are each \( d = 0.25 \) m in depth, and are separated by \( L = 1.75 \) m. The arrangement in Fig. 2-24 occurs at time \( t = 0 \). (a) At what average rate does the layer of people at the door increase? (b) At what time does the layer’s depth reach 5.0 m? (The answers reveal how quickly such a situation becomes dangerous.)

9 In 1 km races, runner 1 on track 1 (with time 2 min, 27.95 s) appears to be faster than runner 2 on track 2 (2 min, 28.15 s). However, length \( L_2 \) of track 2 might be slightly greater than length \( L_1 \) of track 1. How large can \( L_2 - L_1 \) be for us still to conclude that runner 1 is faster?
To set a speed record in a measured (straight-line) distance \(d\), a race car must be driven first in one direction (in time \(t_1\)) and then in the opposite direction (in time \(t_2\)). (a) To eliminate the effects of the wind and obtain the car’s speed \(v\), in a windless situation, should we find the average of \(d/t_1\) and \(d/t_2\) (method 1) or should we divide \(d\) by the average of \(t_1\) and \(t_2\)? (b) What is the fractional difference in the two methods when a steady wind blows along the car’s route and the ratio of the wind speed \(v_w\) to the car’s speed \(v\) is 0.02240?

You are to drive 300 km to an interview. The interview is at 11:15 A.M. You plan to drive at 100 km/h, so you leave at 8:00 A.M. to allow some extra time. You drive at that speed for the first 100 km, but then construction work forces you to slow to 40 km/h for 40 km. What would be the least speed needed for the rest of the trip to arrive in time for the interview?

Traffic shock wave. An abrupt slowdown in concentrated traffic can travel as a pulse, termed a shock wave, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2-25 shows a uniformly spaced line of cars moving at speed \(v = 25.0\) m/s toward a uniformly spaced line of slow cars moving at speed \(v_s = 5.00\) m/s. Assume that each faster car adds length \(L = 12.0\) m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance \(d\) between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?

![Figure 2-25 Problem 12.](image)

You drive on Interstate 10 from San Antonio to Houston, half the time at 55 km/h and the other half at 90 km/h. On the way back you travel half the distance at 55 km/h and the other half at 90 km/h. What is your average speed (a) from San Antonio to Houston, (b) from Houston back to San Antonio, and (c) for the entire trip? (d) What is your average velocity for the entire trip? (e) Sketch \(x\) versus \(t\) for (a), assuming the motion is all in the positive \(x\) direction. Indicate how the average velocity can be found on the sketch.

An electron moving along the \(x\) axis has a position given by \(x = 16t^2 - t^3\), where \(t\) is in seconds. How far is the electron from the origin when it momentarily stops?

(a) If a particle’s position is given by \(x = 4 - 12t + 3t^2\) (where \(t\) is in seconds and \(x\) is in meters), what is its velocity at \(t = 1\) s? (b) Is it moving in the positive or negative direction of \(x\) just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time \(t\); if not, answer no. (f) Is there a time after \(t = 3\) s when the particle is moving in the negative direction of \(x\)? If so, give the time \(t\); if not, answer no.

The position function \(x(t)\) of a particle moving along an \(x\) axis is \(x = 4.0 - 6.0t^2\), with \(t\) in meters and \(t\) in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph \(x\) versus \(t\) for the range \(-5\) s to \(+5\) s. (f) To shift the curve rightward on the graph, should we include the term \(+20t\) or the term \(-20t\) in \(x(t)\)? (g) Does that inclusion increase or decrease the value of \(x\) at which the particle momentarily stops?

The position of a particle moving along the \(x\) axis is given in centimeters by \(x = 9.75 + 1.50t^2\), where \(t\) is in seconds. Calculate (a) the average velocity during the time interval \(t = 2.00\) s to \(t = 3.00\) s; (b) the instantaneous velocity at \(t = 2.00\) s; (c) the instantaneous velocity at \(t = 3.00\) s; (d) the instantaneous velocity at \(t = 2.50\) s; and (e) the instantaneous velocity when the particle is midway between its positions at \(t = 2.00\) s and \(t = 3.00\) s. (f) Graph \(x\) versus \(t\) and indicate your answers graphically.

**Module 2-3 Acceleration**

The position of a particle moving along an \(x\) axis is given by \(x = 12t^2 - 2t^3\), where \(x\) is in meters and \(t\) is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at \(t = 3.0\) s. (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at \(t = 0\))? (i) Determine the average velocity of the particle between \(t = 0\) and \(t = 3\) s.

At a certain time a particle had a speed of 18 m/s in the positive \(x\) direction, and 2.4 s later its speed was 30 m/s in the opposite direction. What is the average acceleration of the particle during this 2.4 s interval?

(a) If the position of a particle is given by \(x = 20t - 5t^3\), where \(x\) is in meters and \(t\) is in seconds, when, if ever, is the particle’s velocity zero? (b) When is its acceleration \(a\) zero? (c) For what time range (positive or negative) is \(a\) negative? (d) Positive? (e) Graph \(x(t)\), \(v(t)\), and \(a(t)\).

From \(t = 0\) to \(t = 5.00\) min, a man stands still, and from \(t = 5.00\) min to \(t = 10.0\) min, he walks briskly in a straight line at a constant speed of 2.20 m/s. What are (a) his average velocity \(\bar{v}\) and (b) his average acceleration \(\bar{a}\) in the time interval 2.00 min to 8.00 min? What are (c) \(v_{avg}\) and (d) \(a_{avg}\) in the time interval 3.00 min to 9.00 min? (e) Sketch \(x\) versus \(t\) and \(v\) versus \(t\), and indicate how the answers to (a) through (d) can be obtained from the graphs.

The position of a particle moving along the \(x\) axis depends on the time according to the equation \(x = ct^2 - bt^3\), where \(x\) is in meters and \(t\) in seconds. What are the units of (a) constant \(c\) and (b) constant \(b\)? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the particle reach its maximum positive \(x\) position? From \(t = 0.0\) s to \(t = 4.0\) s, (d) what distance does the particle move, and (e) what is its displacement? Find its velocity at times (f) 1.0 s, (g) 2.0 s, (h) 3.0 s, and (i) 4.0 s. Find its acceleration at times (j) 1.0 s, (k) 2.0 s, (l) 3.0 s, and (m) 4.0 s.

**Module 2-4 Constant Acceleration**

An electron with an initial velocity \(v_0 = 1.50 \times 10^5\) m/s enters a region of length \(L = 1.00\) cm where it is electrically accelerated (Fig. 2-26). It emerges with \(v = 5.70 \times 10^6\) m/s. What is its acceleration, assumed constant?

Catalyzing mushrooms. Certain mushrooms launch their spores by a catapult mechanism. As water condenses from the air onto a spore that is attached to

![Figure 2-26 Problem 23.](image)
the mushroom, a drop grows on one side of the spore and a film grows on the other side. The spore is bent over by the drop’s weight, but when the film reaches the drop, the drop’s water suddenly spreads into the film and the spores spring upward so rapidly that it is slung off into the air. Typically, the spore reaches a speed of 1.6 m/s in a 5.0 μm launch; its speed is then reduced to zero in 1.0 mm by the air. Using those data and assuming constant accelerations, find the acceleration in terms of g during (a) the launch and (b) the speed reduction.

Figure 2-27 Problems 34 and 35.

Figure 2-27 shows a red car and a green car that move toward each other. Figure 2-28 is a graph of their motion, showing the positions and at time . The green car has a constant speed of 20.0 m/s and the red car begins from rest. What is the acceleration magnitude of the red car?

Figure 2-28 Problem 35.

A certain elevator cab has a total run of 190 m and a maximum speed of 305 m/min, and it accelerates from rest to a constant acceleration equal to 9.8 m/s², which gives the illusion of nor-

Figure 2-29 Problem 37.

A world’s land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. (See Fig. 2-7.) In terms of g, what acceleration did he experience while stopping?

Figure 2-30 Problem 39.

Cars A and B move in the same direction in adjacent lanes. The position of car A is given in Fig. 2-30, from time to . The figure’s vertical scaling is set by . At , car B is at with a velocity of 12 m/s and a negative constant acceleration . (a) What must for such that the cars are (momentarily) side by side (momentarily at the same value of x) at ? (b) For that value of , how many times are the cars side by side? (c) Sketch the position of car B versus time on Fig. 2-30. How many times will the cars be side by side if the magnitude of acceleration is (d) more than and (e) less than the answer to part (a)?

You are driving toward a traffic signal when it turns yellow. Your speed is the legal speed limit of ; your best deceleration rate has the magnitude . Your best reaction time to begin braking is . To avoid having the front of your car enter the intersection after the light turns red, should you brake to a stop or continue to move at 55 km/h if the distance to
the intersection and the duration of the yellow light are (a) 40 m and 2.8 s, and (b) 32 m and 1.8 s? Give an answer of brake, continue, either (if either strategy works), or neither (if neither strategy works and the yellow duration is inappropriate).

As two trains move along a track, their conductors suddenly notice that they are headed toward each other. Figure 2-31 gives their velocities vs time as functions of time t as the conductors slow the trains. The figure’s vertical scaling is set by \( v_i = 40.0 \text{ m/s} \). The slowing processes begin when the trains are 200 m apart. What is their separation when both trains have stopped?

You are arguing over a cell phone while trailing an unmarked police car by 25 m; both your car and the police car are traveling at 110 km/h. Your argument diverts your attention from the police car for 2.0 s (long enough for you to look at the phone and yell, “I won’t do that!”). At the beginning of that 2.0 s, the police officer begins braking suddenly at 5.0 m/s\(^2\). (a) What is the separation between the two cars when your attention finally returns? Suppose that you take another 0.40 s to realize your danger and begin braking. (b) If you too brake at 5.0 m/s\(^2\), what is your speed when you hit the police car?

When a high-speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from a siding and is a distance \( D = 676 \text{ m} \) ahead (Fig. 2-32). The locomotive is moving at 29.0 km/h. The engineer of the high-speed train immediately applies the brakes. (a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume that the engineer is at \( x = 0 \) when, at \( t = 0 \), he first spots the locomotive. Sketch \( x(t) \) curves for the locomotive and high-speed train for the cases in which a collision is just avoided and is not quite avoided.

When startled, an armadillo will leap upward. Suppose it rises 0.544 m in the first 0.200 s. (a) What is its initial speed as it leaves the ground? (b) What is its speed at the height of 0.544 m? (c) How much higher does it go?

(a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? (c) Sketch graphs of \( y, v, \) and \( a \) versus \( t \) for the ball. On the first two graphs, indicate the time at which 50 m is reached.

Raindrops fall 1700 m from a cloud to the ground. (a) If they were not slowed by air resistance, how fast would the drops be moving when they struck the ground? (b) Would it be safe to walk outside during a rainstorm?

At a construction site a pipe wrench struck the ground with a speed of 24 m/s. (a) From what height was it inadvertently dropped? (b) How long was it falling? (c) Sketch graphs of \( y, v, \) and \( a \) versus \( t \) for the wrench.

A hoodlum throws a stone vertically downward with an initial speed of 12.0 m/s from the roof of a building, 30.0 m above the ground. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact?

A hot-air balloon is ascending at the rate of 12 m/s and is 80 m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? (b) With what speed does it hit the ground?

At time \( t = 0 \), apple 1 is dropped from a bridge onto a roadway beneath the bridge; somewhat later, apple 2 is thrown down from the same height. Figure 2-33 gives the vertical positions \( y \) of the apples versus \( t \) during the falling, until both apples have hit the roadway. The scaling is set by \( t_s = 2.0 \text{ s} \). With approximately what speed is apple 2 thrown down?

As a runaway scientific balloon ascends at 19.6 m/s, one of its instrument packages breaks free of a harness and free-falls. Figure 2-34 gives the vertical velocity of the package versus time, from before it breaks free to when it reaches the ground. (a) What maximum height above the break-free point does it rise? (b) How high is the break-free point above the ground?

A bolt is dropped from a bridge under construction, falling 90 m to the valley below the bridge. (a) In how much time does it pass through the last 20% of its fall? What is its speed (b) when it begins that last 20% of its fall and (c) when it reaches the valley beneath the bridge?

A key falls from a bridge that is 45 m above the water. It falls directly into a model boat, moving with constant velocity, that is 12 m from the point of impact when the key is released. What is the speed of the boat?

A stone is dropped into a river from a bridge 43.9 m above the water. Another stone is thrown vertically down 1.00 s after the first is dropped. The stones strike the water at the same time. (a) What is the initial speed of the second stone? (b) Plot velocity versus time on a graph for each stone, taking zero time as the instant the first stone is released.
55 SSM A ball of moist clay falls 15.0 m to the ground. It is in contact with the ground for 20.0 ms before stopping. (a) What is the magnitude of the average acceleration of the ball during the time it is in contact with the ground? ( Treat the ball as a particle.) (b) Is the average acceleration up or down?

56 Figure 2-35 shows the speed $v$ versus height $y$ of a ball tossed directly upward, along a $y$ axis. Distance $d$ is 0.40 m. The speed at height $y_A$ is $v_A$. The speed at height $y_B$ is $\frac{1}{2}v_A$. What is speed $v_A$?

57 To test the quality of a tennis ball, you drop it onto the floor from a height of 4.00 m. It rebounds to a height of 2.00 m. If the ball is in contact with the floor for 12.0 ms, (a) what is the magnitude of its average acceleration during that contact and (b) is the average acceleration up or down?

58 An object falls a distance $h$ from rest. If it travels 0.50$h$ in the last 1.00 s, find (a) the time and (b) the height of its fall. (c) Explain the physically unacceptable solution of the quadratic equation in $t$ that you obtain.

59 Water drips from the nozzle of a shower onto the floor 200 cm below. The drops fall at regular (equal) intervals of time, the first drop striking the floor at the instant the fourth drop begins to fall. When the first drop strikes the floor, how far below the nozzle are the (a) second and (b) third drops?

60 A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

61 A steel ball is dropped from a building’s roof and passes a window, taking 0.125 s to fall from the top to the bottom of the window, a distance of 1.20 m. It then falls to a sidewalk and bounces back past the window, moving from bottom to top in 0.125 s. Assume that the upward flight is an exact reverse of the fall. The time the ball spends below the bottom of the window is 2.00 s. How tall is the building?

62 A basketball player grabbing a rebound jumps 76.0 cm vertically. How much total time (ascent and descent) does the player spend (a) in the top 15.0 cm of this jump and (b) in the bottom 15.0 cm? (The player seems to hang in the air at the top.)

63 A drowsy cat spots a flowerpot that sails first up and then down past an open window. The pot is in view for a total of 0.50 s, and the top-to-bottom height of the window is 2.00 m. How high above the window top does the flowerpot go?

64 A ball is shot vertically upward from the surface of another planet. A plot of $y$ versus $t$ for the ball is shown in Fig. 2-36, where $y$ is the height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure’s vertical scaling is set by $y = 30.0$ m. What are the magnitudes of (a) the free-fall acceleration on the planet and (b) the initial velocity of the ball?

65 Figure 2-15a gives the acceleration of a volunteer’s head and torso during a rear-end collision. At maximum head acceleration, what is the speed of (a) the head and (b) the torso?

66 In a forward punch in karate, the fist begins at rest at the waist and is brought rapidly forward until the arm is fully extended. The speed $v(t)$ of the fist is given in Fig. 2-37 for someone skilled in karate. The vertical scaling is set by $v_f = 8.0$ m/s. How far has the fist moved at (a) time $t = 50$ ms and (b) when the speed of the fist is maximum?

67 When a soccer ball is kicked toward a player and the player deflects the ball by “heading” it, the acceleration of the head during the collision can be significant. Figure 2-38 gives the measured acceleration $a(t)$ of a soccer player’s head for a bare head and a helmeted head, starting from rest. The scaling on the vertical axis is set by $a_r = 200$ m/s$^2$. At time $t = 7.0$ ms, what is the difference in the speed acquired by the bare head and the speed acquired by the helmeted head?

68 A salamander of the genus Hydromantes captures prey by launching its tongue as a projectile: The skeletal part of the tongue is shot forward, unfolding the rest of the tongue, until the outer portion lands on the prey, sticking to it. Figure 2-39 shows the acceleration magnitude $a$ versus time $t$ for the acceleration phase of the launch in a typical situation. The indicated accelerations are $a_2 = 400$ m/s$^2$ and $a_1 = 100$ m/s$^2$. What is the outward speed of the tongue at the end of the acceleration phase?

69 How far does the runner whose velocity–time graph is shown in Fig. 2-40 travel in 16 s? The figure’s vertical scaling is set by $v_f = 8.0$ m/s.
Two particles move along an \( x \) axis. The position of particle 1 is given by \( x = 6.00t^2 + 3.00t + 2.00 \) (in meters and seconds); the acceleration of particle 2 is given by \( a = -8.00t \) (in meters per second squared and seconds) and, at \( t = 0 \), its velocity is 20 m/s. When the velocities of the particles match, what is their velocity?

**Additional Problems**

71 In an arcade video game, a spot is programmed to move across the screen according to \( x = 9.00t - 0.750t^3 \), where \( x \) is distance in centimeters measured from the left edge of the screen and \( t \) is time in seconds. When the spot reaches a screen edge, at either \( x = 0 \) or \( x = 15.0 \text{ cm} \), \( t \) is reset to 0 and the spot starts moving again according to \( x(t) \). (a) At what time after starting is the spot instantaneously at rest? (b) At what value of \( x \) does this occur? (c) What is the spot’s acceleration (including sign) when this occurs? (d) Is it moving right or left just prior to coming to rest? (e) Just after? (f) At what time \( t > 0 \) does it first reach an edge of the screen?

72 A rock is shot vertically upward from the edge of the top of a tall building. The rock reaches its maximum height above the top of the building 1.60 s after being shot. Then, after barely missing the edge of the building as it falls downward, the rock strikes the ground 6.00 s after it is launched. In SI units: (a) with what upward velocity is the rock shot? (b) What maximum height above the top of the building is reached by the rock, and (c) how tall is the building?

73 At the instant the traffic light turns green, an automobile starts with a constant acceleration \( a \) of 2.2 m/s\(^2\). At the same instant a truck, traveling with a constant speed of 9.5 m/s, overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the automobile be traveling at that instant?

74 A pilot flies horizontally at 1300 km/h, at height \( h = 35 \text{ m} \) above initially level ground. However, at time \( t = 0 \), the pilot begins to fly over ground sloping upward at angle \( \theta = 4.3^\circ \) (Fig. 2.41). If the pilot does not change the airplane’s heading, at what time \( t \) does the plane strike the ground?

![Figure 2-41 Problem 74.](image)

75 To stop a car, first you require a certain reaction time to begin braking; then the car slows at a constant rate. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 km/h, and 24.4 m when its initial speed is 48.3 km/h. What are (a) your reaction time and (b) the magnitude of the acceleration?

76 Figure 2-42 shows part of a street where traffic flow is to be controlled to allow a platoon of cars to move smoothly along the street. Suppose that the platoon leaders have just reached intersection 2, where the green appeared when they were distance \( d \) from the intersection. They continue to travel at a certain speed \( v_g \) (the speed limit) to reach intersection 3, where the green appears when they are distance \( d \) from it. The intersections are separated by distances \( D_{12} \) and \( D_{13} \). (a) What should be the time delay of the onset of green at intersection 3 relative to that at intersection 2 to keep the platoon moving smoothly?

Suppose, instead, that the platoon had been stopped by a red light at intersection 1. When the green comes on there, the leaders require a certain time \( t \), to respond to the change and an additional time to accelerate at some rate \( a \) to the cruising speed \( v_g \). (b) If the green at intersection 2 is to appear when the leaders are distance \( d \) from that intersection, how long after the light at intersection 1 turns green should the light at intersection 2 turn green?

77 SSM A hot rod can accelerate from 0 to 60 km/h in 5.4 s. (a) What is its average acceleration, in m/s\(^2\), during this time? (b) How far will it travel during the 5.4 s, assuming its acceleration is constant? (c) From rest, how much time would it require to go a distance of 0.25 km if its acceleration could be maintained at the value in (a)?

78 A red train traveling at 72 km/h and a green train traveling at 144 km/h are headed toward each other along a straight, level track. When they are 950 m apart, each engineer sees the other’s train and applies the brakes. The brakes slow each train at the rate of 0.25 km/s\(^2\). Is there a collision? If so, answer yes and give the separation between the trains when they stop.

79 At time \( t = 0 \), a rock climber accidentally allows a piton to fall freely from a high point on the rock wall to the valley below him. Then, after a short delay, his climbing partner, who is 10 m higher on the wall, throws a piton downward. The positions \( y \) of the pitons versus \( t \) during the falling are given in Fig. 2.43. With what speed is the second piton thrown?

80 A train started from rest and moved with constant acceleration. At one time it was traveling 30 m/s, and 160 m farther on it was traveling 50 m/s. Calculate (a) the acceleration, (b) the time required to travel the 160 m mentioned, (c) the time required to attain the speed of 30 m/s, and (d) the distance moved from rest to the time the train had a speed of 30 m/s. (e) Graph \( x \) versus \( t \) and \( v \) versus \( t \) for the train, from rest.

81 SSM A particle’s acceleration along an \( x \) axis is \( a = 5.0t \), with \( t \) in seconds and \( a \) in meters per second squared. At \( t = 2.0 \text{ s} \), its velocity is +17 m/s. What is its velocity at \( t = 4.0 \text{ s} \)?

82 Figure 2-44 gives the acceleration \( a \) versus time \( t \) for a particle moving along an \( x \) axis. The \( a \)-axis scale is set by \( a_0 = 12.0 \text{ m/s}^2 \). At \( t = -2.0 \text{ s} \), the particle’s velocity is 7.0 m/s. What is its velocity at \( t = 6.0 \text{ s} \)?
A rocket-driven sled running on a straight, level track is used to investigate the effects of large accelerations on humans. One such sled can attain a speed of 1600 km/h in 1.8 s, starting from rest. Find (a) the acceleration (assumed constant) in terms of $g$ and (b) the distance traveled.

A mining cart is pulled up a hill at 20 km/h and then pulled back down the hill at 35 km/h through its original level. (The time required for the cart’s reversal at the top of its climb is negligible.) What is the average speed of the cart for its round trip, from its original level back to its original level?

A motorcyclist who is moving along an $x$ axis directed toward the east has an acceleration given by $a = (6.1 - 1.2t)$ m/s$^2$ for $0 \leq t \leq 6.0$ s. At $t = 0$, the velocity and position of the cyclist are 2.7 m/s and 7.3 m. (a) What is the maximum speed achieved by the cyclist? (b) What total distance does the cyclist travel between $t = 0$ and 6.0 s?

When the legal speed limit for the New York Thruway was increased from 55 mi/h to 65 mi/h, how much time was saved by a motorist who drove the 700 km between the Buffalo entrance and the New York City exit at the legal speed limit?

A car moving with constant acceleration covered the distance between two points 60.0 m apart in 6.00 s. Its speed as it passed the second point was 15.0 m/s. (a) What was the speed at the first point? (b) What was the magnitude of the acceleration? (c) At what prior distance from the first point was the car at rest? (d) Graph $x$ versus $t$ and $v$ versus $t$ for the car, from rest ($t = 0$).

A certain juggler usually tosses balls vertically to a height $H$. To what height must they be tossed if they are to spend twice as much time in the air?

A particle starts from the origin at $t = 0$ and moves along the positive $x$ axis. A graph of the velocity of the particle as a function of the time is shown in Fig. 2.46; the $v$-axis scale is set by $v_x = 4.0$ m/s. (a) What is the coordinate of the particle at $t = 5.0$ s? (b) What is the velocity of the particle at $t = 5.0$ s? (c) What is the acceleration of the particle at $t = 5.0$ s? (d) What is the average velocity of the particle between $t = 1.0$ s and $t = 5.0$ s? (e) What is the average acceleration of the particle between $t = 1.0$ s and $t = 5.0$ s?

A rock is dropped from a 100-m-high cliff. How long does it take to fall (a) the first 50 m and (b) the second 50 m?

Two subway stops are separated by 1100 m. If a subway train accelerates at +1.2 m/s$^2$ from rest through the first half of the distance and decelerates at −1.2 m/s$^2$ through the second half, what are (a) its travel time and (b) its maximum speed? (c) Graph $x$, $v$, and $a$ versus $t$ for the trip.

A stone is thrown vertically upward. On its way up it passes point $A$ with speed $v$, and point $B$, 3.00 m higher than $A$, with speed $\frac{1}{2}v$. Calculate (a) the speed $v$ and (b) the maximum height reached by the stone above point $B$.

A rock is dropped (from rest) from the top of a 60-m-tall building. How far above the ground is the rock 1.2 s before it reaches the ground?

An iceboat has a constant velocity toward the east when a sudden gust of wind causes the iceboat to have a constant acceleration toward the east for a period of 3.0 s. A plot of $x$ versus $t$ is shown in Fig. 2.47, where $t = 0$ is taken to be the instant the wind starts to blow and the positive $x$ axis is toward the east. (a) What is the acceleration of the iceboat during the 3.0 s interval? (b) What is the velocity of the iceboat at the end of the 3.0 s interval? (c) If the acceleration remains constant for an additional 3.0 s, how far does the iceboat travel during this second 3.0 s interval?

A lead ball is dropped in a lake from a diving board 5.20 m above the water. It hits the water with a certain velocity and then sinks to the bottom with this same constant velocity. It reaches the bottom 4.80 s after it is dropped. (a) How deep is the lake? What are the (b) magnitude and (c) direction (up or down) of the average velocity of the ball for the entire fall? Suppose that all the water is drained from the lake. The ball is now thrown from the diving board so that it again reaches the bottom in 4.80 s. What are the (d) magnitude and (e) direction of the initial velocity of the ball?

The single cable supporting an unoccupied construction elevator breaks when the elevator is at rest at the top of a 120-m-high building. (a) With what speed does the elevator strike the ground? (b) How long is it falling? (c) What is its speed when it passes the halfway point on the way down? (d) How long has it been falling when it passes the halfway point?

Two diamonds begin a free fall from rest from the same height, 1.0 s apart. How long after the first diamond begins to fall will the two diamonds be 10 m apart?

A ball is thrown vertically downward from the top of a 36.6-m-tall building. The ball passes the top of a window that is 12.2 m above the ground 2.00 s after being thrown. What is the speed of the ball as it passes the top of the window?
A parachutist bails out and freely falls 50 m. Then the parachute opens, and thereafter she decelerates at 2.0 m/s². She reaches the ground with a speed of 3.0 m/s. (a) How long is the parachutist in the air? (b) At what height does the fall begin?

A ball is thrown down vertically with an initial speed of \( v_0 \) from a height of \( h \). (a) What is its speed just before it strikes the ground? (b) How long does the ball take to reach the ground? What would be the answers to (c) part a and (d) part b if the ball were thrown upward from the same height and with the same initial speed? Before solving any equations, decide whether the answers to (c) and (d) should be greater than, less than, or the same as in (a) and (b).

The sport with the fastest moving ball is jai alai, where measured speeds have reached 200 km/h. Assuming the acceleration is constant, find its magnitude in (a) SI units and (b) in terms of \( g \). (c) How much time \( T_s \) is required for the braking? Your reaction time \( T_r \) is the time you require to perceive an emergency, move your foot to the brake, and begin the braking. If \( T_r = 400 \text{ ms} \), then (d) what is \( T_s \) in terms of \( T_r \), and (e) what is the average time required to stop in a riding or braking? Dark sunglasses delay the visual signals sent from the eyes to the visual cortex in the brain, increasing \( T_r \). (f) In the extreme case in which \( T_r \) is increased by 100 ms, how much farther does the car travel during your reaction time?

In 1889, at Jubbulpore, India, a tug-of-war was finally won after 2 h 41 min, with the winning team displacing the center of the rope 3.7 m. In centimeters per minute, what was the magnitude of the average velocity of that center point during the contest?

Most important in an investigation of an airplane crash by the U.S. National Transportation Safety Board is the data stored on the airplane’s flight-data recorder, commonly called the “black box” in spite of its orange coloring and reflective tape. The recorder is engineered to withstand a crash with an average deceleration of magnitude 3400 g during a time interval of 6.50 ms. In such a crash, if the recorder and airplane have zero speed at the end of that time interval, what is their speed at the beginning of the interval?

From January 26, 1977, to September 18, 1983, George Meegan of Great Britain walked from Ushuaia, at the southern tip of South America, to Prudhoe Bay in Alaska, covering 30 600 km. In meters per second, what was the magnitude of his average velocity during that time period?

The wings on a stonefly do not flap, and thus the insect cannot fly. However, when the insect is on a water surface, it can sail across the surface by lifting its wings into a breeze. Suppose that you time stoneflies as they move at constant speed along a straight path of a certain length. On average, the trips each take 7.1 s with the wings set as sails and 25.0 s with the wings tucked in. (a) What is the ratio of the sailing speed \( v_s \) to the nonsailing speed \( v_{ns} \)? (b) In terms of \( v_s \), what is the difference in the times the insects take to travel the first 2.0 m along the path with and without sailing?

The position of a particle as it moves along a y axis is given by

\[ y = (2.0 \text{ cm}) \sin (\pi t/4), \]

with \( t \) in seconds and \( y \) in centimeters. (a) What is the average velocity of the particle between \( t = 0 \) and \( t = 2.0 \text{ s} \)? (b) What is the instantaneous velocity of the particle at \( t = 0, 1.0, \) and \( 2.0 \text{ s} \)? (c) What is the average acceleration of the particle between \( t = 0 \) and \( t = 2.0 \text{ s} \)? (d) What is the instantaneous acceleration of the particle at \( t = 0, 1.0, \) and \( 2.0 \text{ s} \)?