Review for Exam 2

Exam 2 will cover the following chapters:
- Chapter 7 – Kinetic Energy and Work
- Chapter 8 – Potential Energy and Conservation of Energy
- Chapter 9 – Center of Mass and Linear Momentum
- Chapter 10 – Rotation
- Chapter 11 – Rolling, Torque, and Angular Momentum

There will be 5 problems on Exam 2:
- 4 Problem Set type problems (Choose 3 out of 4 possible problems)
- 1 PLC type ranking question
- 1 Conceptual type question

Chapter 7: Kinetic Energy and Work

Conceptual Type Questions
1. What does (positive, negative, zero) mean?
2. Is work done on an object that moves in a circular path?
3. There are two ways to do the same amount of work – how is that done?
4. If $F_{\text{net}} = 0$, does that mean that there is no work done at all?
5. What role does the dot product have in the definition of work?
6. What does the work-energy theorem mean?
7. How can two objects ($m_1 \neq m_2$) have the same KE?
8. How is KE a (maximum or minimum) in terms of work?
9. Which requires more work: going from 0 m/s to 10 m/s or 20 m/s to 30 m/s?
10. When does the spring force do ("positive" or negative) work? What does this look like on a $F_S$ vs $x$ graph?
11. What can we not use $F = ma$ ($a$ = constant) when a spring force is involved?
12. Why is a spring force call a restoring force?

Calculation Type Problems
1. Work

\[
W = \text{Force along the displacement } \cdot \text{Displacement} = \text{Force } \cdot \text{Displacement along the Force direction} = F \cdot d \cdot \cos \phi
\]

There is positive and negative work:

\[
\begin{align*}
\vec{d} \cdot \vec{F} & \Rightarrow \text{positive work is done} \\
\vec{d} \cdot \vec{F} & \Rightarrow \text{negative work is done}
\end{align*}
\]

There are two types of work:

Work done by a Constant Force – \( W = \vec{F} \cdot d = F_x \Delta x + F_y \Delta y = F \cdot d \cdot \cos \phi \):

Examples:
- Frictional: $W_f = -fd = -\mu Nd$
- Gravitational: $W_g = \pm mgd$
- Applied force: $W_F = \pm F_{\text{app}}d$
Work done by a Variable Force – \( W = \int F \cdot ds \) = area under the \( F \) vs. \( x \) curve:

Examples:
- Spring: \( W_s = \pm \frac{1}{2} kx^2 \)

2. Work-Energy theorem

\[ W = \Delta K = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2 \]

Chapter 8: Potential Energy and Conservation of Energy

Conceptual Type Questions
1. What does it mean to be a conservative force?
2. What types of forces are conservative? Why?
3. What is a PE function?
4. Does it matter which reference I use for PE?
5. What does energy mean physically?
6. Why is energy only meaningful as a difference? In other words, I cannot define the energy at a single point – why?
7. When can conservation of mechanical energy be used?
8. When choosing energy states, what should one look for?
9. How many types of energy are there? Is work considered to be energy as well?
10. When external work is done, is “energy” still conserved?
11. What is nonconservative work? When is it used?

Calculation Type Problems
1. Potential Energy

The PE function \( U(x) \) can be defined if the force doing the work is CONSERVATIVE.

Two conditions:
- \( W_{\text{total}} = \int F_{\text{conservative}} \cdot ds = 0 \)
- Path independent and only depends on the end points.

The potential function is defined in terms of the work-energy theorem:

\[ W = \int_{x_i}^{x_f} F \cdot ds = -\Delta U \rightarrow \Delta U = U(x_f) - U(x_i) = -\int_{x_i}^{x_f} F \cdot ds \text{ or } -\frac{dU(x)}{dx} = F_x \]

Examples:
- \( U_{\text{gravity}} = mgx \)
- \( U_{\text{spring}} = \frac{1}{2} kx^2 \)

2. Conservation of Energy with Nonconservative Work:

\[ E_1 = E_2 + W_{\text{NC}} \]

\[ K_1 + U_1 = K_2 + U_2 + W_{\text{NC}} \]

Energy Diagrams
\[ PE + KE = PE + KE = \text{constant} \]

**Problem Solving Strategies for Conservation of Energy**

Step 1: **Draw** a sketch and **identify** two energy states; labeled them 1 & 2 where one of the energy states is completely known and the other is unknown.

Step 2: **Define** where \( U_g = 0 \) and **identify** what the KE’s, PE’s and the \( W_{NC} \) associated with each of the states. There are 4 types: \( K, U_g, U_S, \) and \( W_{NC} \).

Step 3: **Apply** conservation of energy and solve for the appropriate unknown parameters. If circular motion occurs, Newton’s condition must be applied as well.

Step 4: **Evaluate** your answer. Check whether your answer makes physical sense.

**Important notes:**

1. Typically the way these states are determined is that one of the states is completely known (position and velocity) and the other state has an unknown parameter. As a result, conservation of energy sets up an equation that allows one to solve for the unknown parameter.

2. Newton’s condition on circular motion

   Newton’s 2\(^{nd}\) law places a condition as to whether or not an object actually makes a complete revolution. For example, at the top of the swing, Newton’s 2\(^{nd}\) law says that

   \[
   F_{\text{net}} = \vec{F} + mg = \frac{mv^2}{r} \quad \Rightarrow \quad g = \frac{v^2}{r} = a_c
   \]

   \[
   \Rightarrow \begin{cases} a_c < g, \text{ the ball will not make it to the top} \\ a_c > g, \text{ the ball will have a high speed to make it to the top} \end{cases}
   \]

**Chapter 9: Center of Mass and Linear Momentum**

**Conceptual Type Questions**

1. There are two types of forces for a system \((F_{\text{ext}} \text{ and } F_{\text{int}})\). Why one is the important one – Why?
2. When \( F_{\text{ext}} = 0 \), what happens to \( r_{\text{cm}}, v_{\text{cm}}, \) and \( a_{\text{cm}} \)?
3. When a system is in (equilibrium or nonequilibrium), what does it mean?
4. What is momentum physically?
5. How can two objects \((m_1 \neq m_2)\) have the same momentum?
6. What does impulse physically mean for collisions?
7. How does one get a (large or small) applied force during a collision?
8. When is momentum conserved? When is it not?
9. How is \( v_{\text{cm}} \) and momentum physically connected to each other?
10. What does it mean to be (elastic or inelastic) for a collision?
11. Can one use conservation of energy during a collision – why?
12. When are the elastic collision equations valid?
13. In reading a problem, how do you check to see if the collision is elastic or inelastic?

**Calculation Type Problems**

1. **Impulse-Momentum Theorem**

   \[
   \vec{J} = \int_{t_0}^{t_f} \vec{F} \, dt = \vec{F}_{\text{ave}} \cdot \Delta t = \Delta \vec{p}
   \]

   For constant change in momentum,

   \[
   \Delta \vec{p} = \text{constant} = \vec{F}_{\text{ave}} \cdot \Delta t = \vec{F}_{\text{ave}} \cdot \Delta t
   \]
2. Conservation of Momentum

**Inelastic collisions:** \( \vec{P}_0 = \vec{P}_f, \quad K_0 \neq K_f \)

- Completely inelastic: \( \vec{p}_{10} + \vec{p}_{20} = \vec{p}_i \Rightarrow m_f \vec{v}_{10} + m_2 \vec{v}_{20} = (m_f + m_2)\vec{v}_i \)
- Partially inelastic: \( \vec{p}_{10} + \vec{p}_{20} = \vec{p}_{i1} + \vec{p}_{2f} \Rightarrow m_f \vec{v}_{10} + m_2 \vec{v}_{20} = m_f \vec{v}_{i1} + m_2 \vec{v}_{2f} \)

**Elastic collisions:** \( \vec{P}_0 = \vec{P}_i, \quad K_0 = K_f \)

Must use the elastic collision equations:

\[
\begin{align*}
\vec{v}_{i1} &= \frac{m_1 - m_2}{m_1 + m_2} \vec{v}_{10} + \frac{2m_2}{m_1 + m_2} \vec{v}_{20} \\
\vec{v}_{2f} &= \frac{2m_1}{m_1 + m_2} \vec{v}_{10} + \frac{m_2 - m_1}{m_1 + m_2} \vec{v}_{20}
\end{align*}
\]

**Problem Solving Strategies for Conservation of Linear Momentum**

1. **Are external forces** acting on the system? Typically, if the collision force is much larger than the external force acting on the system, the external force can be ignored and conservation of momentum can be applied. However, if an external force cannot be ignored, momentum is not a conserved quantity and the impulse-momentum theorem must be applied instead.

2. **Identify** the type of collision (elastic or inelastic); setup a “before and after” picture, and only **apply** conservation of momentum during the collision process. Remember that 99.99% of all collision does not conserved energy and therefore, cannot be applied during the collision.

   - **Elastic:** \( \vec{P}_0 = \vec{P}_i \) & \( K_0 = K_i \) \( \Rightarrow \vec{p}_{10} + \vec{p}_{20} = \vec{p}_{i1} + \vec{p}_{2f} \) \( \Rightarrow K_{10} + K_{20} = K_{i1} + K_{2f} \)
   - **Inelastic:** \( \vec{P}_0 = \vec{P}_i \) & \( K_0 \neq K_i \) \( \Rightarrow \vec{p}_{10} + \vec{p}_{20} = \vec{p}_{i1} + \vec{p}_{2f} \) \( \Rightarrow K_{10} + K_{20} \neq K_{i1} + K_{2f} \)
   - **Completely Inelastic:** \( \vec{P}_0 = \vec{P}_i \) & \( K_0 \neq K_i \) \( \Rightarrow \vec{p}_{10} + \vec{p}_{20} = \vec{p}_i \) \( \Rightarrow K_{10} + K_{20} \neq K_{i1} + K_{2f} \)

3. **Apply** conservation of energy before or after the collision to find additional information about the system.

4. **Evaluate** your answer. Check whether your answer makes physical sense.

3. **Center of Mass**

\[
\begin{align*}
\bar{r}_{com} &= \frac{1}{M} \sum_i m_i \bar{r}_i \\
\vec{P}_{total} &= M \vec{v}_{com} \\
\sum F = 0 \Rightarrow & \begin{cases} 
\text{static equilibrium:} & (\vec{r}_{cm})_0 = (\vec{r}_{cm})_i \\
\text{dynamic equilibrium:} & (\vec{v}_{cm})_0 = (\vec{v}_{cm})_i 
\end{cases}
\end{align*}
\]
Chapter 10: Rotation
Conceptual Type Questions

Calculation Type Problems

1. Rotational Kinematics Equations

<table>
<thead>
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<th>Data</th>
<th>( \Delta \theta )</th>
<th>( \alpha )</th>
<th>( \omega )</th>
<th>( \omega_0 )</th>
<th>( t )</th>
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<td>( \omega = \omega_0 + \alpha t )</td>
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<td>( \Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2 )</td>
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2. Translational and Rotational quantities

\( s = r \theta \)  
\( v = r \omega \)  
\( a_t = r \alpha \)  
\( a_r = \frac{v^2}{r} = \omega^2 r \)

3. Rotational Inertia

Parallel-Axis theorem:

\( I_{\text{off-axis}} = I_{\text{cm}} + Mh^2 \)

where \( h \) is the distance from the CM axis of rotation to the off-axis location.

4. Newton’s 2\(^{\text{nd}}\) Law

\[ \tau = rF = rF_{\perp} = rF \sin \theta \quad \Rightarrow \quad \sum F = ma \]
\[ \sum \tau = I \alpha \]
\[ a = r \alpha \]

Chapter 11: Rolling, Torque, and Angular Momentum

Conceptual Type Questions

Calculation Type Problems

1. Conservation of Energy with the rolling condition

\[
E_1 = E_2 \\
(U_g + K)_1 = (U_g + K)_2 \\
(U_g + \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2)_1 = (U_g + \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2)_2|_{\text{rolling condition}}
\]

2. Right-Hand Rule

\[ \vec{\tau} = \vec{r} \times \vec{F} \quad \vec{\omega} = \vec{r} \times \vec{p} \quad \vec{\tau} = \frac{d\vec{L}}{dt} \]

3. Conservation of Angular Momentum

\[
\begin{align*}
L_{\text{point}} &= r \vec{p} = r \vec{p}_{\perp} = mrv \sin \theta \\
L_{\text{rigid body}} &= I \omega \\
\end{align*}
\]
\[
L_0 = L_i \\
(L_i + L_2 + \cdots)_0 = (L_i + L_2 + \cdots)_f
\]