Chapter 24: Electric Potential
Example Questions & Problems

\[ \Delta V = \frac{-W}{q} = \frac{\Delta U}{q} \quad V = -\int \mathbf{E} \cdot d\mathbf{s} \quad V_{\text{point}} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \quad \mathbf{E} = -\frac{\partial V}{\partial s} \]

Example 24.1
a. If a negative charge is initially at rest in an electric field, will it move toward a region of higher potential or lower potential? What about a positive charge? How does the potential energy of the charge change in each of these two instances?
b. If the electric field \( \mathbf{E} \) is uniform in a region, what can you infer about the electric potential \( V \)? If \( V \) is uniform in a region of space, what can you infer about \( \mathbf{E} \)?
c. The earth has a net electric charge that causes an electric field at points near its surface equal 150 N/C and directed in toward the center of the earth. Why would it still be possible to adopt the earth’s surface as a standard reference point of potential and to assign the potential \( V = 0 \) to it?
d. How can you ensure that the electric potential in a given region of space will have the same value through that region?

Example 24.2
An electric dipole consists of two point charges, \( q_1 = -q_2 = 12 \text{nC} \), placed 10 cm apart. (a) Compute the potentials at points A, B, and C. (b) Plot \( V(r) \) vs. \( r \) and interpret the plot.

Example 24.3
The figure shows electric potential \( V(x) \) along an x axis. A proton moves inside the potential well. (a) Interpret the motion of the proton and calculate the force (magnitude and direction) as it moves (i) left of \( x = 3.0 \text{ cm} \) and (ii) right of \( x = 5.0 \text{ cm} \). (b) The proton is released at \( x = 3.5 \text{ cm} \) with initial kinetic energy 4.00 eV going left. What is the x coordinate and potential value of its turning point? (c) If instead the proton moves to the right with the same kinetic energy, what is the speed at \( x = 6.0 \text{ cm} \)?
Example 24.4
Two isolated, concentric, conducting spherical shells of negligible thicknesses have radii and charge as shown. (a) Plot $E(r)$ and $V(r)$ before you start your calculations. Set $V(\text{ref} = \infty) = 0$. (b) Calculate the electric field at radial distances $r = 4.00\text{m}, 0.70\text{m},$ and $0.20\text{m}$? (c) Calculate the potentials $V(r)$ at radial distances $r = 4.00\text{m}, 1.00\text{m}, 0.70\text{m}, 0.50\text{m}, 0.20\text{m}$ and $r = 0$?
Example A
A graph of the \( x \) component of the electric field as a function of \( x \) in a region of space is shown in the figure. The scale of the vertical axis is set by \( E_{xS} = 20.0 \text{ N/C} \). The \( y \) and \( z \) components of the electric field are zero in this region. If the electric potential at the origin is 10 V, (a) what is the electric potential at \( x = 2.0 \text{ m} \), (b) what is the greatest positive value of the electric potential for points on the \( x \) axis for which \( 0 \leq x \leq 6.0 \text{ cm} \), and (c) for what value of \( x \) is the electric potential zero?

Solution
a. By Eq. 24-18, the change in potential is the negative of the “area” under the curve. Thus, using the area-of-a-triangle formula, we have
\[
V - 10 = - \int_{0}^{x=2} E \cdot ds = \frac{1}{2} \cdot 2 \cdot 20 \rightarrow V = 30 \text{ V}
\]
b. For any region within \( 0 < x < 3 \text{ m} \), \(-\int E \cdot ds\) is positive, but for any region for which \( x > 3 \text{ m} \) it is negative. Therefore, \( V = V_{\text{max}} \) occurs at \( x = 3 \text{ m} \).
\[
V_{\text{max}} - 10 = - \int_{0}^{x=3} E \cdot ds = \frac{1}{2} \cdot 3 \cdot 20 \rightarrow V_{\text{max}} = 40 \text{ V}
\]
c. In view of our result in part (b), we see that now (to find \( V = 0 \)) we are looking for some \( X > 3 \text{ m} \) such that the “area” from \( x = 3 \text{ m} \) to \( x = X \) is 40 V. Using the formula for a triangle (\( 3 < x < 4 \)) and a rectangle (\( 4 < x < X \)), we require
\[
\frac{1}{2} \cdot 1 \cdot 20 + 20 \cdot (X - 4) = 40 \rightarrow X = 5.5 \text{ m}
\]

Example B
Two tiny metal spheres \( A \) and \( B \) of mass \( m_A = 5.00 \text{ g} \) and \( m_B = 10.0 \text{ g} \) have equal positive charge \( q = 5.00 \mu \text{C} \). The spheres are connected by a massless nonconducting string of length \( d = 1.00 \text{ m} \), which is much greater than the radii of the spheres. (a) What is the electric potential energy of the system? (b) Suppose you cut the string. At that instant, what is the acceleration of each sphere? (c) A long time after you cut the string, what is the speed of each sphere?

Solution
a. The potential energy is
\[
U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.0 \times 10^{-6} \text{ C})^2}{1.00 \text{ m}} = 0.225 \text{ J} = U
\]
relative to the potential energy at infinite separation.
b. Each sphere repels the other with a force that has magnitude
\[
F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.0 \times 10^{-6} \text{ C})^2}{(1.00 \text{ m})^2} = 0.225 \text{ N}.
\]
According to Newton’s second law the acceleration of each sphere is the force divided by the mass of the sphere. Let \( m_A \) and \( m_B \) be the masses of the spheres. The acceleration of sphere \( A \) and \( B \) is
\[
a_A = \frac{F}{m_A} = \frac{0.225 \text{ N}}{5.0 \times 10^{-3} \text{ kg}} = 45.0 \text{ m/s}^2 = a_A \quad \text{and} \quad a_B = \frac{F}{m_B} = \frac{22.5 \text{ m/s}^2}{m_B} = a_B
\]
c. Energy and momentum is conserved.

Conservation of energy
The initial potential energy is \( U_0 = 0.225 \text{ J} \), as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is \( \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 \), where \( v_A \) and \( v_B \) are the final velocities. Thus,
Conservation of momentum

\[ P_0 = P_f \rightarrow 0 = m_A v_A + m_B v_B \]

solving for \( v_B \)

\[ v_B = -\frac{m_A}{m_B} v_A \]

Substituting \( v_B \) from the momentum equation into the energy equation, and collecting terms, we obtain

\[ U_0 = \frac{1}{2} (m_A / m_B) (m_A + m_B) v_A^2. \]

Therefore,

\[ v_A = \sqrt{\frac{2U_m}{m_A (m_A + m_B)}} = \sqrt{\frac{2(0.225 \text{ J})(10 \times 10^{-3} \text{ kg})}{(5.0 \times 10^{-3} \text{ kg})(5.0 \times 10^{-3} \text{ kg} + 10 \times 10^{-3} \text{ kg})}} = 7.75 \text{ m/s} = v_A \]

and

\[ v_B = -\frac{m_A}{m_B} v_A - \left( \frac{5.0 \times 10^{-3} \text{ kg}}{10 \times 10^{-3} \text{ kg}} \right) (7.75 \text{ m/s}) = -3.87 \text{ m/s} = v_B \]

Example C

Two metal spheres, each of radius 3.0 cm, have a center-to-center separation of 2.0 m. Sphere 1 has charge \(+1.0 \times 10^{-8} \text{ C}\); sphere 2 has charge \(-3.0 \times 10^{-8} \text{ C}\). Assume that the separation is large enough for us to assume that the charge on each sphere is uniformly distributed (the spheres do not affect each other). With \( V = 0 \) at infinity, calculate (a) the potential at the point halfway between the centers and the potential on the surface of (b) sphere 1 and (c) sphere 2.

Solution

a. The electric potential is the sum of the contributions of the individual spheres. Let \( q_1 \) be the charge on one, \( q_2 \) be the charge on the other, and \( d \) be their separation. The point halfway between them is the same distance \( d/2 \) (= 1.0 m) from the center of each sphere, so the potential at the halfway point is

\[ V = \frac{q_1 + q_2}{4\pi \varepsilon_0 d/2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{1.0 \text{ m}} \cdot \left(1.0 \times 10^{-8} \text{ C} - 3.0 \times 10^{-8} \text{ C}\right) \]

\[ = -1.8 \times 10^2 \text{ V} = V \]

b. The distance from the center of one sphere to the surface of the other is \( d - R \), where \( R \) is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

\[ V_1 = \frac{1}{4\pi \varepsilon_0} \left[ \frac{q_1}{R} + \frac{q_2}{d-R} \right] = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[ \frac{1.0 \times 10^{-8} \text{ C}}{0.030 \text{ m}} - \frac{3.0 \times 10^{-8} \text{ C}}{2.0 \text{ m} - 0.030 \text{ m}} \right] \]

\[ = 2.9 \times 10^7 \text{ V} = V_1 \]

c. The potential at the surface of sphere 2 is

\[ V_2 = \frac{1}{4\pi \varepsilon_0} \left[ \frac{q_1}{d-R} + \frac{q_2}{R} \right] = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[ \frac{1.0 \times 10^{-8} \text{ C}}{2.0 \text{ m} - 0.030 \text{ m}} - \frac{3.0 \times 10^{-8} \text{ C}}{0.030 \text{ m}} \right] \]

\[ = -8.9 \times 10^7 \text{ V} = V_2 \]