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**The Biot–Savart Law**  The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law. This law asserts that the contribution \(d\vec{B}\) to the field produced by a current-length element \(i \, ds\) at a point \(P\) located a distance \(r\) from the current element is

\[
d\vec{B} = \frac{\mu_0 i \, ds \times \hat{r}}{4\pi r^2} \quad \text{(Biot–Savart law).} \quad (29-3)
\]

Here \(\hat{r}\) is a unit vector that points from the element toward \(P\). The quantity \(\mu_0\), called the permeability constant, has the value

\[4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \approx 1.26 \times 10^{-6} \text{T} \cdot \text{m/A}.\]

**Magnetic Field of a Long Straight Wire**  For a long straight wire carrying a current \(i\), the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance \(R\) from the wire,

\[B = \frac{\mu_0 i}{2\pi R} \quad \text{(long straight wire).} \quad (29-4)\]

**Magnetic Field of a Circular Arc**  The magnitude of the magnetic field at the center of a circular arc, of radius \(R\) and central angle \(\phi\) (in radians), carrying current \(i\), is

\[B = \frac{\mu_0 i \phi}{4\pi R} \quad \text{(at center of circular arc).} \quad (29-9)\]

**Force Between Parallel Currents**  Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on one length \(L\) of either wire is

\[F_{ba} = i_a L B_a \sin 90^\circ = \frac{\mu_0 i_a L j_b}{2 \pi d}, \quad \text{(29-13)}\]

where \(d\) is the wire separation, and \(i_a\) and \(i_b\) are the currents in the wires.

**Ampere’s Law**  Ampere’s law states that

\[\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere’s law).} \quad (29-14)\]

The line integral in this equation is evaluated around a closed loop called an Amperean loop. The current \(i\) on the right side is the net current encircled by the loop. For some current distributions, Eq. 29-14 is easier to use than Eq. 29-3 to calculate the magnetic field due to the currents.

**Fields of a Solenoid and a Toroid**  Inside a long solenoid carrying current \(i\), at points not near its ends, the magnitude \(B\) of the magnetic field is

\[B = \mu_0 i n \quad \text{(ideal solenoid),} \quad (29-23)\]

where \(n\) is the number of turns per unit length. At a point inside a toroid, the magnitude \(B\) of the magnetic field is

\[B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \quad \text{(toroid),} \quad (29-24)\]

where \(r\) is the distance from the center of the toroid to the point.

**Field of a Magnetic Dipole**  The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point \(P\) located a distance \(z\) along the coil’s perpendicular central axis is parallel to the axis and is given by

\[\vec{B}(z) = \frac{\vec{\mu}}{2\pi z}, \quad \text{(29-27)}\]

where \(\vec{\mu}\) is the dipole moment of the coil. This equation applies only when \(z\) is much greater than the dimensions of the coil.

**QUESTIONS**

1. Figure 29-23 shows three circuits, each consisting of two radial lengths and two concentric circular arcs, one of radius \(r\) and the other of radius \(R > r\). The circuits have the same current through them and the same angle between the two radial lengths. Rank the circuits according to the magnitude of the net magnetic field at the center, greatest first.

![](image-a.png)

Fig. 29-23  Question 1.

2. Figure 29-24 represents a snapshot of the velocity vectors of four electrons near a wire carrying current \(i\). The four velocities have the same magnitude; velocity \(\vec{v}_2\) is directed into the page. Electrons 1 and 2 are at the same distance from the wire, as are electrons 3 and 4. Rank the electrons according to the magnitudes of the magnetic forces on them due to current \(i\), greatest first.

![](image-b.png)

Fig. 29-24  Question 2.

3. Figure 29-25 shows four arrangements in which long parallel wires...
carry equal currents directly into or out of the page at the corners of identical squares. Rank the arrangements according to the magnitude of the net magnetic field at the center of the square, greatest first.

4 Figure 29-26 shows cross sections of two long straight wires; the left-hand wire carries current $i_1$ directly out of the page. If the net magnetic field due to the two currents is to be zero at point $P$, (a) should the direction of current $i_2$ in the right-hand wire be directly into or out of the page and (b) should $i_2$ be greater than, less than, or equal to $i_1$?

5 Figure 29-27 shows three circuits consisting of straight radial lengths and concentric circular arcs (either half- or quarter-circles of radii $r$, $2r$, and $3r$). The circuits carry the same current. Rank them according to the magnitude of the magnetic field produced at the center of curvature (the dot), greatest first.

6 Figure 29-28 gives, as a function of radial distance $r$, the magnitude of the magnetic field inside and outside four wires ($a$, $b$, $c$, and $d$), each of which carries a current that is uniformly distributed across the wire's cross section. Overlapping portions of the plots are indicated by double labels. Rank the wires according to (a) radius, (b) the magnitude of the magnetic field on the surface, and (c) the value of the current, greatest first. (d) Is the magnitude of the current density in wire $a$ greater than, less than, or equal to that in wire $c$?

7 Figure 29-29 shows four circular Amperian loops ($a$, $b$, $c$, $d$) concentric with a wire whose current is directed out of the page. The current is uniform across the wire's circular cross section (the shaded region). Rank the loops according to the magnitude of $\int B \cdot ds$ around each, greatest first.

8 Figure 29-30 shows four arrangements in which long, parallel, equally spaced wires carry equal currents directly into or out of the page. Rank the arrangements according to the magnitude of the net force on the central wire due to the currents in the other wires, greatest first.

9 Figure 29-31 shows four circular Amperian loops ($a$, $b$, $c$, $d$) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of the page, 9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of $\int B \cdot ds$ around each, greatest first.

10 Figure 29-32 shows four identical currents $i$ and five Amperian paths ($a$ through $e$) encircling them. Rank the paths according to the value of $\int B \cdot ds$ taken in the directions shown, most positive first.

11 Figure 29-33 shows three arrangements of three long straight wires carrying equal currents directly into or out of the page. (a) Rank the arrangements according to the magnitude of the net force on wire $A$ due to the currents in the other wires, greatest first. (b) In arrangement 3, is the angle between the net force on wire $A$ and the dashed line equal to, less than, or more than 45°?
sec. 29-2 Calculating the Magnetic Field Due to a Current

•1 A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A. (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth’s magnetic field at the site is 20 μT.

•2 Figure 29-34a shows an element of length ds = 1.00 μm in a very long straight wire carrying current. The current in that element sets up a differential magnetic field dB at points in the surrounding space. Figure 29-34b gives the magnitude dB of the field for points 2.5 cm from the element, as a function of angle θ between the wire and a straight line to the point. The vertical scale is set by dB0 = 60.0 pT. What is the magnitude of the magnetic field set up by the entire wire at perpendicular distance 2.5 cm from the wire?

•3 SSM At a certain location in the Philippines, Earth’s magnetic field of 39 μT is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?

•4 A straight conductor carrying current i = 5.0 A splits into identical semicircular arcs as shown in Fig. 29-35. What is the magnetic field at the center C of the resulting circular loop?

•5 In Fig. 29-36, a current i = 10 A is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius R = 5.0 mm. Point b is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of B at a and (c) magnitude and (d) direction of B at b?

•6 In Fig. 29-37, point P is at perpendicular distance R = 2.00 cm from a very long straight wire carrying a current. The magnetic field B set up at point P is due to contributions from all the identical current-elements ds along the wire. What is the distance x to the element making (a) the greatest contribution to field B and (b) 100% of the greatest contribution?

•7 In Fig. 29-38, two circular arcs have radii a = 13.5 cm and b = 10.7 cm, subtend angle θ = 74.0°, carry current i = 0.411 A, and share the same center of curvature P. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at P?

•8 In Fig. 29-39, two semicircular arcs have radii R0 = 7.80 cm and R1 = 3.15 cm, carry current i = 0.281 A, and share the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C?

•9 SSM Two long straight wires are parallel and 8.0 cm apart. They are to carry equal currents such that the magnetic field at a point halfway between them has magnitude 300 μT. (a) Should the currents be in the same or opposite directions? (b) How much current is needed?

•10 In Fig. 29-40, a wire forms a semicircle of radius R = 9.26 cm and two (radial) straight segments each of length L = 13.1 cm. The wire carries current i = 34.8 mA. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle’s center of curvature C?

•11 In Fig. 29-41, two long straight wires are perpendicular to the page and separated by distance d1 = 0.75 cm. Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point P located at distance d2 = 1.50 cm from wire 2?

•12 In Fig. 29-42, two long straight wires at separation d = 16.0 cm carry currents i1 = 3.61 mA and i2 = 3.00i1 out of the page. (a) Where on the x axis is the net magnetic field equal to zero? (b) If the two currents are doubled, is the zero-field point shifted toward wire 1, shifted toward wire 2, or unchanged?
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**13** In Fig. 29-43, point P₁ is at distance \( R = 13.1 \) cm on the perpendicular bisector of a straight wire of length \( L = 18.0 \) cm carrying current \( i = 58.2 \) mA. (Note that the wire is not long.) What is the magnitude of the magnetic field at \( P₁ \) due to \( i \)?

**14** Equation 29-4 gives the magnitude \( B \) of the magnetic field set up by a current in an *ininitely long* straight wire. (a) Using Eq. 29-4 to calculate \( B \) then results in a certain percentage error. What value must the ratio \( L/R \) exceed if the percentage error is to be less than 1.00%? That is, what \( L/R \) gives \( (B \text{ from Eq. 29-4}) - (B \text{ actual}) \) \( (B \text{ actual}) \) (100%) = 1.00%?

**15** Figure 29-44 shows two current segments. The lower segment carries a current of \( i₁ = 0.40 \) A and includes a semicircular arc with radius 5.0 cm, angle 180°, and center point \( P \). The upper segment carries current \( i₂ = 2i₁ \) and includes a circular arc with radius 4.0 cm, angle 120°, and the same center point \( P \). What are the (a) magnitude and (b) direction of the net magnetic field \( \vec{B} \) at \( P \) for the indicated current directions? What are the (c) magnitude and (d) direction of \( \vec{B} \) if \( i₁ \) is reversed?

**16** In Fig. 29-45, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop 1 has radius 1.50 cm and carries 4.00 mA. Loop 2 has radius 2.50 cm and carries 6.00 mA. Loop 2 is to be rotated about a diameter while the net magnetic field \( \vec{B} \) set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is 100 nT?

**17** In Fig. 29-43, point \( P₂ \) is at perpendicular distance \( R = 25.1 \) cm from one end of a straight wire of length \( L = 13.6 \) cm carrying current \( i = 0.693 \) A. (Note that the wire is not long.) What is the magnitude of the magnetic field at \( P₂ \)?

**18** A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm, a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29-46a shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is 47.25 \( \mu \)T. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-46b). The magnetic field produced at the (same) center of curvature now has magnitude 15.75 \( \mu \)T, and its direction is reversed. What is the radius of the smaller semicircle?

**19** One long wire lies along an \( x \) axis and carries a current of 30 A in the positive \( x \) direction. A second long wire is perpendicular to the \( xy \) plane, passes through the point (0, 4.0 m, 0), and carries a current of 40 A in the positive \( z \) direction. What is the magnitude of the resulting magnetic field at the point (0, 2.0 m, 0)?

**20** In Fig. 29-47, part of a long insulated wire carrying current \( i = 5.78 \) mA is bent into a circular section of radius \( R = 1.89 \) cm. In unit-vector notation, what is the magnetic field at the center of curvature \( C \) if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated 90° counterclockwise as indicated?

**21** Figure 29-48 shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance \( d₁ = 6.00 \) m and distance \( d₂ = 4.00 \) m. What is the magnitude of the net magnetic field at point \( P \), which lies on a perpendicular bisector to the wires?

**22** Figure 29-49a shows, in cross section, two long, parallel wires carrying current and separated by distance \( L \). The ratio \( i₁/i₂ \) of their currents is 4.00; the directions of the currents are not indicated. Figure 29-49b shows the \( y \) component \( B_y \) of their net magnetic field along the \( x \) axis to the right of wire 2. The vertical scale is set by \( B_{y,\text{max}} = 4.0 \) nT, and the horizontal scale is set by \( x_s = 200 \) cm. (a) At what value of \( x > 0 \) is \( B_y \) maximum? (b) If \( i₂ = 3 \) mA, what is the value of that maximum? What is the direction (into or out of the page) of (c) \( i₁ \) and (d) \( i₂ \)?

**23** Figure 29-50 shows a snapshot of a proton moving at velocity \( \vec{v} = (-200 \text{ m/s}) \parallel \) toward a long straight wire with current \( i = 350 \) mA. At the instant shown, the proton’s distance from the wire is \( d = 2.89 \) cm. In unit-vector notation, what is the magnetic force on the proton due to the current?

**24** Figure 29-51 shows, in cross section, four thin wires that are parallel, straight, and very long. They carry identical currents in the directions indicated. Initially all four wires are at distance \( d = 15.0 \) cm from the origin of the coordinate system, where they cre-
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In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 8.50$ cm. Each wire carries 15.0 A, and all the currents are out of the page. In unit-vector notation, what is the net magnetic force per meter of wire length on wire 1?

In Fig. 29-65, a long straight wire carries a current $i_1 = 30.0$ A and a rectangular loop carries current $i_2 = 20.0$ A. Take $a = 1.00$ cm, $b = 8.00$ cm, and $L = 30.0$ cm. In unit-vector notation, what is the net force on the loop due to $i_2$?

sec. 29-4 Ampere's Law

In a particular region there is a uniform current density of 15 A/m² in the positive $z$ direction. What is the value of $\mathbf{B} \cdot d\mathbf{s}$ when that line integral is calculated along the three straight-line segments from $(x, y, z)$ coordinates $(a, b, c)$ to $(d, 2b, c)$ to $(d, 0, c)$ to $(a, 0, c)$, where $d = 20$ cm?

Figure 29-66 shows a cross section across a diameter of a long cylindrical conductor of radius $a = 2.00$ cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) $r = 0$, (b) $r = 1.00$ cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?

Figure 29-67 shows two closed paths wrapped around two conducting loops carrying currents $i_1 = 5.0$ A and $i_2 = 3.0$ A. What is the value of the integral $\mathbf{B} \cdot d\mathbf{s}$ for (a) path 1 and (b) path 2?

Each of the eight conductors in Fig. 29-68 carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\mathbf{B} \cdot d\mathbf{s}$. What is the value of the integral for (a) path 1 and (b) path 2?

Eight wires cut the page perpendicularly at the points shown in Fig. 29-69. A wire labeled with the integer $k$ ($k = 1, 2, \ldots, 8$) carries the current $i_k$, where $i = 4.50$ mA. For those wires with odd $k$, the current is out of the page; for those with even $k$, it is into the page. Evaluate $\mathbf{B} \cdot d\mathbf{s}$ along the closed path in the direction shown.

The current density $\mathbf{j}$ inside a long, solid, cylindrical wire of radius $a = 3.1$ mm is in the direction of the central axis, and its magnitude varies linearly with radial distance $r$ from the axis according to $J = j_0/r$, where $j_0 = 310$ A/m². Find the magnitude of the magnetic field at (a) $r = 0$, (b) $r = a/2$, and (c) $r = a$.

In Fig. 29-70, a long circular pipe with outside radius $R = 2.6$ cm carries a (uniformly distributed) current $i = 8.00$ mA into the page. A wire runs parallel to the pipe at a distance of 3.00R from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point $P$ has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

**sec. 29-5 Solenoids and Toroids**

A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A. (It is made up of a square solenoid—instead of a round one as in Fig. 29-16—bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?

A solenoid that is 95.0 cm long has a radius of 2.00 cm and a winding of 1200 turns; it carries a current of 3.60 A. Calculate the magnitude of the magnetic field inside the solenoid.

A 200-turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.29 A. Calculate the magnitude of the magnetic field $\mathbf{B}$ inside the solenoid.

A solenoid 1.30 m long and 2.60 cm in diameter carries a current of 18.0 A. The magnetic field inside the solenoid is 23.0 mT. Find the length of the wire forming the solenoid.

A long solenoid has 100 turns/cm and carries current $i$. An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is $0.0460c$ ($c =$ speed of light). Find the current $i$ in the solenoid.

An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is 800 m/s and its velocity vector makes an angle of $30^\circ$ with the central axis of the solenoid. The solenoid carries 4.0 A and has 8000 turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid’s opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)

A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA. A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at $45.0^\circ$ to the axial direction? (b) What is the magnitude of the magnetic field there?

**sec. 29-6 A Current-Carrying Coil as a Magnetic Dipole**

Figure 29-71 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius...
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Three long wires are parallel to a z-axis, and each carries a current of 10 A in the positive z direction. Their points of intersection with the xy plane form an equilateral triangle with sides of 50 cm, as shown in Fig. 29-77. A fourth wire (wire b) passes through the midpoint of the base of the triangle and is parallel to the other three wires. If the net magnetic force on wire a is zero, what are the (a) size and (b) direction (±z or −z) of the current in wire b?

Figure 29-78 shows a closed loop with current i = 2.00 A. The loop consists of a half-circle of radius 4.00 m, two quarter-circles each of radius 2.00 m, and three radial straight wires. What is the magnitude of the net magnetic field at the common center of the circular sections?

A 10-gauge bare copper wire (2.6 mm in diameter) can carry a current of 50 A without overheating. For this current, what is the magnitude of the magnetic field at the surface of the wire?

A long vertical wire carries an unknown current. Coaxial with the wire is a long, thin, cylindrical conducting surface that carries a current of 30 mA upward. The cylindrical surface has a radius of 3.0 mm. If the magnitude of the magnetic field at a point 5.0 mm from the wire is 1.0 μT, what are the (a) size and (b) direction of the current in the wire?

Figure 29-79 shows a cross section of a long cylindrical conductor of radius a = 4.00 cm containing a long cylindrical hole of radius b = 1.50 cm. The central axes of the cylinder and hole are parallel and are distance d = 2.00 cm apart; current i = 5.25 A is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases b = 0 and d = 0.

The magnitude of the magnetic field 88.0 cm from the axis of a long straight wire is 7.30 μT. What is the current in the wire?

Figure 29-80 shows a wire segment of length Δx = 3.0 cm, centered at the origin, carrying current i = 2.0 A in the positive y direction (as part of some complete circuit). To calculate the magnitude of the magnetic field B produced by the segment at a point several meters from the origin, we can use $B = (\mu_0/4\pi)i \Delta x (\sin \theta)/r^2$ as the Biot–Savart law. This is because r and θ are essentially constant over the segment. Calculate B (in unit-vector notation) at the (x, y, z) coordinates (a) (0, 0, 5.0 m), (b) (0, 6.0 m, 0), (c) (7.0 m, 7.0 m, 0), and (d) (−3.0 m, −4.0 m, 0).

Figure 29-81 shows, in cross section, two long parallel wires spaced by distance d = 10.0 cm; each carries 100 A, out of the page in wire 1. Point P is on a perpendicular bisector of the line connecting the wires. In unit-vector notation, what is the net magnetic field at P if the current in wire 2 is (a) out of the page and (b) into the page?

In Fig. 29-82, two infinitely long wires carry equal currents i. Each follows a 90° arc on the circumference of the same circle of radius R. Show that the magnetic field B at the center of the circle is the same as the field B a distance R below an infinite straight wire carrying a current i to the left.

A long wire carrying 100 A is perpendicular to the magnetic field lines of a uniform magnetic field of magnitude 5.0 mT. At what distance from the wire is the net magnetic field equal to zero?

A long, hollow, cylindrical conductor (with inner radius 2.0 mm and outer radius 4.0 mm) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is co-axial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm, (b) 3.0 mm, and (c) 5.0 mm from the central axis of the wire and cylinder?

A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire?

Figure 29-83 shows a cross section of an infinite conducting sheet carrying a current per unit x-length of λ; the current emerges perpendicularly out of the page. (a) Use the Biot–Savart law and symmetry to show that for all points P above the sheet and all points P’ below it, the magnetic field B is parallel to the sheet and directed as shown. (b) Use Ampere’s law to prove that $B = \mu_0/2\pi \lambda$ at all points P and P’.

Figure 29-84 shows, in cross section, two parallel wires that are separated by distance $d = 18.6$ cm. Each carries 4.23 A, out of the page in wire 1 and into the page in wire 2. In unit-vector notation, what is the net magnetic field at point P at distance $R = 34.2$ cm, due to the two currents?
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83 SSM In unit-vector notation, what is the magnetic field at point P in Fig. 29-85 if \( i = 10 \) A and \( a = 8.0 \) cm? (Note that the wires are not long.)

84 Three long wires all lie in an xy plane parallel to the x axis. They are spaced equally, 10 cm apart. The two outer wires each carry a current of 5.0 A in the positive x direction. What is the magnitude of the force on a 3.0 m section of either of the outer wires if the current in the center wire is 3.2 A (a) in the positive x direction and (b) in the negative x direction?

85 SSM Figure 29-86 shows a cross section of a hollow cylindrical conductor of radii \( a \) and \( b \), carrying a uniformly distributed current \( i \). (a) Show that the magnetic field magnitude \( B(r) \) for the radial distance \( r < b < a \) is given by

\[
B = \frac{\mu_0 i}{2\pi (a^2 - b^2)} \frac{r^2 - b^2}{r}.
\]

(b) Show that when \( r = a \), this equation gives the magnetic field magnitude \( B \) at the surface of a long straight wire carrying current \( i \); when \( r = b \), it gives zero magnetic field; and when \( b = 0 \), it gives the magnetic field inside a solid conductor of radius \( a \) carrying current \( i \). (c) Assume that \( a = 2.0 \) cm, \( b = 1.8 \) cm, and \( i = 100 \) A, and then plot \( B(r) \) for the range \( 0 < r < 6 \) cm.

86 Show that the magnitude of the magnetic field produced at the center of a rectangular loop of wire of length \( L \) and width \( W \), carrying a current \( i \), is

\[
B = \frac{2\mu_0 i}{\pi} \frac{(L^2 + W^2)^{1/2}}{LW}.
\]

87 Figure 29-87 shows a cross section of a long conducting coaxial cable and gives its radii \( (a, b, c) \). Equal but opposite currents \( i \) are uniformly distributed in the two conductors. Derive expressions for \( B(r) \) with radial distance \( r \) in the ranges (a) \( r < c \), (b) \( c < r < b \), (c) \( b < r < a \), and (d) \( r > a \). (e) Test these expressions for all the special cases that occur to you. (f) Assume that \( a = 2.0 \) cm, \( b = 1.8 \) cm, \( c = 0.40 \) cm, and \( i = 120 \) A and plot the function \( B(r) \) over the range \( 0 < r < 3 \) cm.

88 Figure 29-88 is an idealized schematic drawing of a rail gun. Projectile \( P \) sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let \( w \) be the distance between the rails, \( R \) the radius of each rail, and \( i \) the current. (b) Show that the force on the projectile is directed to the right along the rails and is given approximately by

\[
F = \frac{i\mu_0}{2\pi} \ln \frac{w + R}{R}.
\]

(b) If the projectile starts from the left end of the rails at rest, find the speed \( v \) at which it is expelled at the right. Assume that \( i = 450 \) kA, \( w = 12 \) mm, \( R = 6.7 \) cm, \( L = 4.0 \) m, and the projectile mass is 10 g.

89 A square loop of wire of edge length \( a \) carries current \( i \). Show that, at the center of the loop, the magnitude of the magnetic field produced by the current is

\[
B = \frac{2\sqrt{2\mu_0 i}}{ma}.
\]

90 In Fig. 29-71, an arrangement known as Helmholtz coils consists of two circular coaxial coils, each of \( N \) turns and radius \( R \), separated by distance \( s \). The two coils carry equal currents \( i \) in the same direction. (a) Show that the first derivative of the magnitude of the net magnetic field of the coils \( (dB/dx) \) vanishes at the mid-point \( P \) regardless of the value of \( s \). Why would you expect this to be true from symmetry? (b) Show that the second derivative \( (d^2B/dx^2) \) also vanishes at \( P \), provided \( s = R \). This accounts for the uniformity of \( B \) near \( P \) for this particular coil separation.

91 SSM A square loop of wire of edge length \( a \) carries current \( i \). Show that the magnitude of the magnetic field produced at a point on the central perpendicular axis of the loop and a distance \( x \) from its center is

\[
B(x) = \frac{4\mu_0 i a^2}{\pi(4x^2 + a^2)(4x^2 + 2a^2)^{1/2}}.
\]

Prove that this result is consistent with the result shown in Problem 89.

92 Show that if the thickness of a toroid is much smaller than its radius of curvature (a very skinny toroid), then Eq. 29-24 for the field inside a toroid reduces to Eq. 29-23 for the field inside a solenoid. Explain why this result is to be expected.

93 SSM Show that a uniform magnetic field \( \vec{B} \) cannot drop abruptly to zero (as is suggested by the lack of field lines to the right of point \( a \) in Fig. 29-89) as one moves perpendicular to \( \vec{B} \), say along the horizontal arrow in the figure. (Hint: Apply Ampere’s law to the rectangular path shown by the dashed lines.) In actual magnets, “fringing” of the magnetic field lines always occurs, which means that \( \vec{B} \) approaches zero in a gradual manner. Modify the field lines in the figure to indicate a more realistic situation.

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