Problem Set 1
Due: see website for due dates
Chapter 1: The Special Theory of Relativity (Tipler)

Question A
When Albert Einstein was a boy of 16, he mulled over the following puzzler: A runner looks at herself in a mirror that she holds at arm’s length in front of her. If she runs with nearly the speed of light, will she be able to see herself in the mirror? Analyze this question using the Principle of Relativity.

Question B
A worried student writes, “I still cannot believe your solution to the Twin Paradox. During the outward trip to Canopus (300 cy from earth), each twin can regard the other as moving away from him; so how can we say which twin is younger? The answer is that the twin in the rocket makes a turn, and in Lorentz spacetime geometry, the greatest aging is experienced by the person who does not turn. This argument is extremely unsatisfying. It forces me to ask: what if the rocket breaks down when I get to “Canopus, so that I stop there but cannot turn around? Does this mean that it is no longer possible to say that I have aged less than my earthbound twin? But if not, then I would never have gotten to Canopus alive.” Write a response to this student, answering the questions politely and decisively.

Question C
Imagine a thought experiment were you are riding your bike along Pacific Garden Mall (downtown Santa Cruz) and the speed of light is 5 m/s (≈10 mph) instead of 3.00 x 10^8 m/s. How would everyday life appear? That is, what would be the appearance of (i) a person standing on the sidewalk or the windows in the storefronts appear as you ride by (ii) A second person riding a bike that is accelerating to catch up to you? (iii) The clock tower at the end of the street? Explain your answer in detail.

Question D
Suppose we have two masses particles (A and B) facing each other and both are traveling at the speed of light. If an observer jumps into the frame of particle A, then my point of view of particle B’s velocity is c + c = 2c, double the speed of light. Why is this thought experiment no valid? Hint: think about how particle A measures time.

Question E
Our goal is to prove: Clocks synchronized in one reference frame are not, in general, synchronized in another inertial frame moving relative to the first.
An astronaut takes two sources of pulsed light. Both sources are placed at the midpoint of the craft. One is directed towards the front of the spacecraft, and the other towards the rear. They point at targets placed at equal distances from their respective source. The two sources each emit a pulse at the exact same instant.

a. How do the pulses arrival times compare as seen from the perspective of the astronaut? Now plot spacetime diagram showing this?

b. But what does the mission controller (on earth) conclude when he observes what is going on in the spacecraft as it speeds past him going left? Now plot spacetime diagram showing this?
Problem 1
Muons are subatomic particles that are produced several miles above the earth's surface as a result of collisions of cosmic rays (charged particles, such as protons, that enter the earth's atmosphere from space) with atoms in the atmosphere. These muons rain down more-or-less uniformly on the ground, although some of them decay on the way since the muon is unstable with proper half-life of about 1.5 μs. In a certain experiment a muon detector is carried in a balloon to an altitude of 2000 m, and in the course of 1 hour it registers 650 muons traveling at 0.99c toward the earth. If an identical detector remains at sea level, how many muons would you expect it to register in 1 hour? (Remember that after n half-lives the number of muons surviving from an initial sample of $N_0$ is $N_0/2^n$, and don’t forget about time dilation.) This was essentially the method used in the first tests of time dilation, starting in the 1940’s.

Problem 2
Suppose that an event occurs in inertial frame S with coordinates $x = 75$ m, $y = 18$ m, $z = 4.0$ m at $t = 2.0 \times 10^{-5}$ s. The inertial frame S’ moves in the +x direction with $v = 0.85c$. The origins of S and S’ coincided at $t = t’ = 0$. (a) What are the coordinates of the event in S’? (b) Use the inverse transformation on the results of (a) to obtain the original coordinates.

Problem 3
Observers in a frame S arrange for two simultaneous explosions at time $t = 0$. The first explosion is at the origin ($x_1 = y_1 = z_1 = 0$) while the second is on the positive x axis 4 light years away ($x_2 = 4 \text{ cly}$, $y_2 = z_2 = 0$). (a) Use the Lorentz transformation to find the coordinates of these two events as observed in a frame S’ traveling in the standard configuration at speed 0.6c relative to S. (b) How far apart are the two events as measured in S’? (c) Are the events simultaneous as observed in S’?

Problem 4
A friend of yours who is the same age as you travels to the star Alpha Centauri, which is 4 cly away, and returns immediately. He claims that the entire trip took just 6 years. (a) How fast did he travel? (b) How old are you when he returns? (c) Draw a spacetime diagram that verifies your answer to (a) and (b).

Problem 5
Two spaceships, each 100 m long when measured at rest, travel toward each other with speeds of 0.85c relative to earth. (a) How long is each ship as measured by someone on Earth? (b) How fast is each ship traveling as measured by an observer on the other? (c) At one time $t = 0$ on earth, the fronts of the ships are together as they just begin to pass each other. At what time on earth are their ends together? (d) Sketch accurately scaled diagrams in the frame of one of the ships showing the passing of the other ship.

Problem 6
A meter stick is moving with speed 0.8c relative to a frame S. (a) What is the stick’s length, as measured by observers in S, if the stick is parallel to its velocity v? (b) What if the stick is perpendicular to v? (c) What if the stick is at 60° to v, as seen in the stick’s rest frame? [Hint: You can imagine that the meter-stick is the hypotenuse of a 30-60-90 triangle of plywood.] (d) What if the stick is at 60° to v, as measured in S?

Problem 7
A group of $\pi$ mesons (pions) is observed traveling at speed 0.8c in a particle-physics laboratory. (a) If the pions proper half-life is $1.8 \times 10^{-8}$ s, what is their half-life as observed in the lab frame? (b) If there were initially 32,000 pions, how many will be left after they have traveled 36 m? (c) What would be the answer to (c) if one ignored time
dilation? (d) Now consider the view of the pions’ rest frame. In part (b) how far (as seem by the pions) does the laboratory move, and how long does this take? How many pions remain at the end of this time?

Problem 8
Consider a tale of the physicist who is ticketed for running a red light and argues that, because was approaching the intersection, the red light was Doppler shifted and appeared green. How fast would he have been going? ($\lambda_{\text{red}} = 650 \text{ nm} & \lambda_{\text{green}} = 530 \text{ nm}$)

Problem 9
A baseball at rest has such a stupendous amount of energy ($mc^2$ and more in Chapter 2) because it is effectively in motion through spacetime at the speed of light. What is troubling to most people is the fact that the baseball is travelling through spacetime at the speed of light even though it is at rest relative to “our” frame.

a. So why is the baseball moving at the speed of light through spacetime? Hint: (i) Starting from the spacetime position 4-vector, calculate the magnitude of the 4-velocity $u = \frac{dx}{d\tau}$. (ii) Now show that the speed of the baseball in the rest frame of the observer has a speed of c.

b. Derive that square of the 4-velocity square in part (a) can be written as

$$u^2 = c^2 = \frac{c^2 dt^2 - dx^2}{dt^2} \quad \text{rewritten as} \quad c^2 \left( \frac{dt}{d\tau} \right)^2 + \left( \frac{dx}{dt} \right)^2 = c^2$$

Now interpret this relationship that an increase in the baseball’s speed through space must be accompanied by a decrease in the baseball’s speed through time.

Problem 10
A worried student writes, "relativity must be wrong. Consider a 20 meter long pole carried by a runner so fast in the direction of its length that it appears to be only 10 meters long in the rest frame of the earth. This pole can evidently be enclosed in a barn 10 meters long! However, from the point of view of the runner carrying the pole the barn appears to be contracted to half its length, i.e. 5 meters. Does not this unbelievable conclusion probe that relativity contains somewhere a fundamental logical inconsistency?"

a. How can a 20 meter pole fit in a 5 meter barn? How fast is the runner running? Provide a clear resolution of this paradox in words.

b. Draw a spacetime diagram from the barn's point of view, showing clearly the worldlines of the two ends of the pole, the two ends of the barn, and the simultaneity line corresponding to the barn-moment the pole is enclosed in the barn.

c. Draw another spacetime diagram, this one from the runner's point of view, showing the same things.