Exam 2 Equation Sheet

\[ i_C = C \frac{dv_C}{dt}, \quad v_L = L \frac{di_L}{dt}, \quad v_C(0^+) = v_C(0^+), \quad i_L(0^+) = i_L(0^+) \]

**Constant Source Circuits**

RC-Circuit: \( v_C(t) = V_{oc} + [v_C(0^+) - V_{oc}] e^{-\frac{t}{\tau}} \) where \( \tau = R_{th} C_{eq} \) and \( V_{oc} = v_{th} \)

RL-Circuit: \( i_L(t) = I_{sc} + [i_L(0^+) - I_{sc}] e^{-\frac{t}{\tau}} \) where \( \tau = \frac{L_{eq}}{R_{th}} \) and \( I_{sc} = i_n \)

**Time-Dependent Source Circuits**

Step 1: Use KVL or KCL to derive the first order equation \( dx(t)/dt + ax(t) = f(t) \)

Step 2: Determine the complete response \( x(t) = x_n + x_f \)

- Pick-off the \( 1/\tau \) term and write \( x_n = Ae^{-\frac{t}{\tau}} \)
- The forcing response has the same functional form as the effective source:
  \[ x_f(t) = \begin{cases} B_1 + B_2 e^{-\frac{t}{\tau}} & \text{for } B_1 + B_2 < 0 \\ B_1 + B_2 \sin(\omega t) + B_3 \cos(\omega t) & \text{for } B_1 + B_2 > 0 \end{cases} \]

Step 3: Determine constants (i) \( (B_1, B_2) \), (ii) \( x(0^-) \) & (iii) \( A \)

- Substitute the force response into the first order equation and determine \( (B_1, B_2) \).
- Solve the circuit at \( t = 0^- \) to determine the boundary condition \( x(0^-) \).
- Apply the BC \( x(0^-) \) to the complete response to determine \( A \).

Step 4: Solve the circuit problem and plot and interpret the complete response.

**Second Order Circuits**

Step 1: Use KVL, KCL, \( v_L = L \dot{\hat{s}} i_L \), and \( i_C = C \hat{s} v_C \) to obtain two first order response equations that are functions of \( v_C \) and \( i_L \). Solve for the 2\(^{nd}\) order equation and determine the force response \( B \).

Step 2: Set the effective source equal to zero and obtain the characteristic equation \& roots:

\[ s^2 + 2\alpha s + \omega_0^2 = 0 \quad \rightarrow \quad s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \]

From the values of \( \alpha \) and \( \omega_0 \), determine the type of natural response of the circuit.

- If \( \alpha = \omega_0 \) → Critically Damped: \( x_{\text{critical}}(t) = e^{-\alpha t}(A_1 t + A_2) \)
- If \( \alpha > \omega_0 \) → Overdamped: \( x_{\text{over}}(t) = A_1 e^{\alpha t} + A_2 e^{\alpha t} \)
- If \( \alpha < \omega_0 \) → Underdamped:
  \[ x_{\text{under}}(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t), \quad \text{where} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} \]

- Write out the complete response: \( x(t) = A_1 x_{n1} + A_2 x_{n2} + B \)

Step 3: Determine \( v_C(0^-), i_L(0^-) \) and use equations (1) \& (2) in Step 1 to numerically solve for the derivative \( \hat{s} v_C(0^-) \) after the switch has been thrown.

Step 4: Determine the constants \( A_1 \) and \( A_2 \) by applying the boundary conditions to the complete response solution by

- Setting up two equations involving \( x_C(0^-) \) and \( dx_C(0^-)/dt \) and solve for \( A_1 \) and \( A_2 \)
- Write down the complete response for the circuit.

Step 5: Solve the circuit problem and interpret

\[ \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad \text{solution} \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix} \begin{bmatrix} D_2 \\ D_2 \end{bmatrix} \]

where \( D_2 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - b_1 a_2 \)