Please show all work - even work done by calculator.

1. Fast Auto Service provides oil and lube service for cars. It is known that the mean time taken for oil and lube service at this garage is 15 minutes per car and the standard deviation is 2.4 minutes. The management wants to promote the business by guaranteeing a maximum waiting time for its customers. If a customer's car is not serviced within that period, the customer will receive a 50% discount on the charges. The company wants to limit this discount to at most 5% of the customers. What should the maximum guaranteed waiting time be? Assume that the times taken for oil and lube service for all cars have a normal distribution.

\[ X = \mu + Z \sigma \]
\[ X = 15 + 1.65(2.4) \]
\[ X = 18.98 \]

Max guaranteed wait time should be about 18 min (at 5% significance).

2. A consumer agency surveyed all 2500 families living in a small town to collect data on the number of television sets owned by them. The following table lists the frequency distribution of the data collected by this agency.

<table>
<thead>
<tr>
<th>Number of TV sets owned</th>
<th>Number of Families</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>1</td>
<td>970</td>
</tr>
<tr>
<td>2</td>
<td>730</td>
</tr>
<tr>
<td>3</td>
<td>410</td>
</tr>
<tr>
<td>4</td>
<td>270</td>
</tr>
</tbody>
</table>

a. Construct the probability distribution table for the number of TV sets owned by these families.

\[
\begin{array}{c|c}
 X & P(x) \\
0 & 120/2500 = .048 \\
1 & 970/2500 = .388 \\
2 & 730/2500 = .292 \\
3 & 410/2500 = .164 \\
4 & 270/2500 = .108 \\
\end{array}
\]

b. Let \( x \) denote the number of TV sets owned by a randomly selected family from this town. Find the probability that \( P(x > 3) \).

\[
P(x > 3) = P(x = 4) = .108
\]
3. According to Smith Travel Research, the mean charges for a hotel room in the United States were $84.58 per day in 2009. Assume that the current hotel room rates have a normal distribution with a mean of $84.58 per day and a standard deviation of $12. Find the percentage of hotel rooms with rates between $75 and $92.

\[ z = \frac{x - \mu}{\sigma} \]

For \( x = 75 \):
\[ z = \frac{75 - 84.58}{12} = -0.798 \]
Look up, area = 0.2119

For \( x = 92 \):
\[ z = \frac{92 - 84.58}{12} = 0.618 \]
Look up, get area = 0.7324

\[ 0.7324 - 0.2119 = 0.5205 \]
52.05% have rates between $75 and $92.

4. According to CardWeb.com, the average credit card debt per household was $8367 in 2009. Assume that the probability distribution of all such current debts is skewed to the right with a mean of $8367 and a standard deviation of $2400. Find the probability that the mean of a random sample of 225 such debts is between $8100 and $8500.

Large sample \( n \geq 30 \) so by Central Limit Theorem for means, sampling dist is normally distr:

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2400}{\sqrt{225}} = 160 \]

For \( x = 8100 \):
\[ z = \frac{8100 - 8367}{160} = -1.67 \]
Look up, area = 0.0475

For \( x = 8500 \):
\[ z = \frac{8500 - 8367}{160} = 0.83 \]
Look up, get area = 0.7967

\[ 0.7967 - 0.0475 = 0.7492 \]
74.9% chance that the mean of a random sample of 225 such debts is in that range.
5. During hard economic times, people switch between brands while shopping and rarely stick to one brand. According to an Insight Express online survey of shoppers, only 24% faithfully buy a favorite cereal. Assume that this percentage is true for the current population of all shoppers. Using the binomial formula, find the probability that in a random sample of 10 shoppers, exactly 4 faithfully buy the same cereal.

\[
P(X) = \binom{n}{x} p^x q^{n-x}
\]

\[
= \binom{10}{4} (0.24)^4 (0.76)^6
\]

\[
= 210 \times (0.24)^4 \times (0.76)^6 = 0.1343
\]

6. In a Gallup Poll of adults, 80% of the respondents said that they feel stress “frequently” or “sometimes” in their daily lives. Assume that this percentage is true for the current population of all adults. Using the binomial probabilities table, find the probability that the number of adults in a random sample of 15 who feel stress frequently or sometimes is at most 9.

\[
x = 0, 1, 2, ..., 7, 8, 9
\]

\[
\binom{n}{x}
\times
\begin{array}{c}
0.0000 + 0.0000 + 0.0000 + 0.0000 + 0.0000 + 0.0007 + 0.0035 + 0.0138 + 0.0430
\end{array}
\]

\[
= 0.0611
\]

7. Magnetic resonance imaging (MRI) is a process that produces internal body images using a strong magnetic field. Some patients become claustrophobic and require sedation because they are required to lie within a small, enclosed space during the MRI test. Suppose that 20% of all patients undergoing MRI testing require sedation due to claustrophobia. If five patients are selected at random, find the probability that the number of patients in these five who require sedation is exactly 2.

\[
P(X) = \binom{n}{x} p^x q^{n-x}
\]

\[
= \binom{5}{2} (0.20)^2 (0.80)^3
\]

\[
= \frac{5!}{2!3!} \times (0.20)^2 \times (0.80)^3 = 0.2048
\]
8. The management of a supermarket wants to adopt a new promotional policy of giving a free gift to every customer who spends more than a certain amount per visit at this supermarket. The expectation of the management is that after this promotional policy is advertised, the expenditures for all customers at this supermarket will be normally distributed with a mean of $95 and a standard deviation of $21. If the management decides to give free gifts to all those customers who spend more than $130 at this supermarket during a visit, what percentage of the customers are expected to get free gifts?

\[
\begin{align*}
\frac{x - \mu}{\sigma} &= \frac{130 - 95}{21} \\
&= 1.67 \\
\text{Look up, area = .9525} \\
\text{So area in right tail} \\
&= 1 - .9525 = .0475 \\
\text{4.75% get free gifts}
\end{align*}
\]

9. The GPAs of all 5540 students enrolled at a university have an approximately normal distribution with a mean of 3.02 and a standard deviation of 0.29. Let \( \bar{x} \) be the mean GPA of a random sample of 48 students selected from this university. Find the mean and standard deviation of \( \bar{x} \) and comment on the shape of its sampling distribution.

\[
\begin{align*}
\mu_{\bar{x}} &= \mu = 3.02 \\
\sigma_{\bar{x}} &= \frac{\sigma}{\sqrt{n}} = \frac{0.29}{\sqrt{48}} = 0.042
\end{align*}
\]

Sampling dist is approx normal
by the Central Limit Theorem
since the sample size is large \( n \geq 30 \)
10. Mong Corporation makes auto batteries. The company claims that 80% of its LL-70 auto batteries are good for 70 months or longer. Assume that this claim is true. Let \( \hat{p} \) be the proportion in a sample of 100 such batteries that are good for 70 months or longer. What is the probability that this sample proportion is between 0.75 and 0.85?

\[
\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.80 \cdot 0.20}{100}} = 0.04
\]

For \( \hat{p} = 0.75 \)

\[
z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.75 - 0.80}{0.04} = -1.25
\]

Look up, get area = 0.1056

For \( \hat{p} = 0.85 \)

\[
z = \frac{0.85 - 0.80}{0.04} = 1.25
\]

Look up, get area = 0.8944

0.8944 - 0.1056 = 0.7888

78.88% chance that this sample proportion is between 0.75 and 0.85.