Instructions: Please show your work in the space provided and clearly mark your answers. Remember to include units where appropriate and simplify all answers, unless otherwise stated.

1. (7 pts) Consider the curve determined by \( \mathbf{r}(t) = (4t^2, 2t^3 - 4, 3t + 2) \). Find the vector equation (or parametric equations) of the line tangent to the curve at the point \((16, 12, 8)\).

\[
\mathbf{r}'(t) = \langle 8t, 6t^2, 3 \rangle \\
\mathbf{r}'(2) = \langle 16, 24, 3 \rangle
\]

\[
\mathbf{r}(2) = \langle 16, 12, 8 \rangle
\]

\[
x = 16 + 16t \\
y = 12 + 24t \\
z = 8 + 3t
\]

2. (7 pts) Use differentials to estimate the amount of tin used to make a closed cylindrical tin can that is 22 ft tall and 8 ft in diameter if the top and bottom are 0.8 ft thick and the sides are 0.04 ft thick.

\[
V = \pi r^2 h \\
\begin{align*}
\frac{dV}{dt} &= 2\pi rh \frac{dr}{dt} + \pi r^2 \frac{dh}{dt} \\
\frac{dV}{dh} &= 2\pi r \frac{dr}{dt} + \pi r^2 \frac{dr}{dh} \\
\end{align*}
\]

\[
\begin{align*}
dr &= 0.04 \\
dh &= 0.8 \\
\frac{dr}{dh} &= \frac{1}{16} \\
dV &= 102.5 \\
\text{About } 102.5 \text{ ft}^3
\end{align*}
\]
3. (7 pts) Find the parametric equations (or the vector equation) of the curve formed by the intersection of the surfaces $y^2 + z^2 = 36$ and $z = 6 - x$.

Let $y = 6 \cos t$, $z = 6 \sin t \implies x = 6 - z$

then $\mathbf{r}(t) = \langle 6 - 6 \sin t, 6 \cos t, 6 \sin t \rangle$

4. (10 pts) Stranded on Mars, Mark Watney hurls a potato in frustration. Realizing he needs to eat, he decides he should look for his wayward potato. He estimates that he threw the potato due north at an angle of $30^\circ$ with the horizontal, with an initial speed of 15 m/s, and from a height of 2 m. Gravity on Mars produces a downward acceleration of 3.71 m/s$^2$. There was also a crosswind blowing due east that he estimates would produce an acceleration of 7 m/s$^2$ on the potato. What are the coordinates of the point where he should begin looking for his potato?

\[ \mathbf{a} = \langle 7, 0, -3.71 \rangle \]
\[ \mathbf{v}(t) = \langle 7t + C_1, C_2, -3.71t + C_3 \rangle \]

\[ \mathbf{v}(t) = \langle 7t, \frac{15\sqrt{3}}{2}, -3.71t + \frac{15}{2} \rangle \]

\[ \mathbf{r}(t) = \langle \frac{7}{2}t^2 + C_1, \frac{15\sqrt{3}}{2}t + C_2, C_3 \rangle \]

\[ \mathbf{v}(t) = \langle 3.5t^2, 12.99 + \sqrt{1.855t^2 + 7.3t + 2} \rangle \]

Solve $-1.855t^2 + 7.3t + 2 = 0$
\[ t \approx 4.298 \]

\[ \mathbf{r}(4.29) \approx \langle 64.41, 55.73, 0 \rangle \]
5. Let $\mathbf{r}(t) = 2t^2 \mathbf{i} + \frac{2}{3} t^3 \mathbf{j} + 4t \mathbf{k}$

(a) (6 pts) Find the length of the curve from $t = 0$ to $t = 2.$

$$\mathbf{r}'(t) = \langle 4t, 2t^2, 4 \rangle$$

$$|\mathbf{r}'(t)| = \sqrt{4t^4 + 16t^2 + 16} = 2(t^2 + 2)$$

$$2 \int_0^2 t^2 + 2 \, dt = 2 \left[ \frac{t^3}{3} + 2t \right]_0^2 = 2 \left[ \frac{8}{3} + 4 \right] = 2 \left( \frac{16}{3} \right)$$

$$= 2 \left( \frac{16}{3} \right) = \frac{40}{3}$$

(b) (10 pts) Find $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ at $t = 0$.

$$\mathbf{T} = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{t^2 + 2} \langle 2t, t^2, 2 \rangle$$

$$\mathbf{T}(0) = \langle 0, 0, 1 \rangle$$

$$\mathbf{N} = \frac{\mathbf{T}'}{|\mathbf{T}'|} = \frac{\mathbf{T}'}{t^2 + 2} = \frac{2}{t^2 + 2} \langle 2t, t^2, 2 \rangle + \frac{1}{t^2 + 2} \langle 2, 2t, 0 \rangle$$

$$\mathbf{N}(0) = \langle 1, 0, 0 \rangle \quad |\mathbf{N}(0)| = 1$$

So $\mathbf{N}(0) = \langle 1, 0, 0 \rangle$

$$\mathbf{B} = \mathbf{T} \times \mathbf{N}$$

$$\mathbf{B}(0) = \langle 0, 1, 0 \rangle$$
6. (8 pts) Find the following limit, or prove that it does not exist.

\[
\lim_{{(x,y)\to(0,0)}} \frac{-4x^6 y \cos(xy)}{3x^{12} + 5y^2}
\]

Along the x-axis (y = 0):

\[
\lim_{{(x,0)\to(0,0)}} \frac{-4x^6 \cos(0)}{3x^{12} + 0} = 0
\]

Along the line y = x^6:

\[
\lim_{{(x,x^6)\to(0,0)}} \frac{-4x^{12} \cos(x^6)}{3x^{12} + 5x^{12}} = -\frac{1}{2}
\]

Different limits so limit \( \text{DNE} \)

7. (8 pts) Sketch the domain of the function \( f(x, y) = \sqrt{4 - x^2} + \ln(49 - x^2 - y^2) \).

A: \( 4 - x^2 \geq 0 \)
\[
\begin{align*}
A &= \{ x | -2 \leq x \leq 2 \} \\
B &= \{ y | 0 < y < 49 \}
\end{align*}
\]

B: \( 49 - x^2 - y^2 > 0 \)
\[
\begin{align*}
\text{Interior of circle } r = 7
\end{align*}
\]

8. (7 pts) Find the equation of the plane tangent to the surface \( f(x, y) = x + \sin(y/x) \) at the point (3, 0)

\[
f(3,0) = 3 = z_0
\]

\[
f_x = 1 + \frac{y}{x^2} \cos \left( \frac{y}{x} \right)
\]

\[
f_x(3,0) = 1
\]

\[
f_y = \frac{1}{x} \cos \left( \frac{y}{x} \right)
\]

\[
f_y(3,0) = \frac{1}{3}
\]

\[
z - z_0 = f_x(3,0) (x-3) + f_y(3,0) (y-0)
\]

\[
z - 3 = 1(x-3) + \frac{1}{3} (y-0)
\]

\[
z - 3 = x - 3 + \frac{1}{3} y
\]

\[
x + \frac{1}{3} y - 2 = 0
\]