**Brewster’s Law**

For light incident at the Brewster angle $\theta_B$, we find experimentally that the reflected and refracted rays are perpendicular to each other. Because the reflected ray is reflected at the angle $\theta_B$ in Fig. 33-25 and the refracted ray is at an angle $\theta_r$, we have

$$\theta_B + \theta_r = 90^\circ.$$  \hspace{1cm} (33-46)

These two angles can also be related with Eq. 33-40. Arbitrarily assigning subscript 1 in Eq. 33-40 to the material through which the incident and reflected rays travel, we have, from that equation,

$$n_1 \sin \theta_B = n_2 \sin \theta_r.$$  \hspace{1cm} (33-47)

Combining these equations leads to

$$n_1 \sin \theta_B = n_2 \sin(90^\circ - \theta_B) = n_2 \cos \theta_B,$$  \hspace{1cm} (33-48)

which gives us

$$\theta_B = \tan^{-1} \frac{n_2}{n_1} \quad \text{(Brewster angle).}$$  \hspace{1cm} (33-49)

(Note carefully that the subscripts in Eq. 33-49 are not arbitrary because of our decision as to their meanings.) If the incident and reflected rays travel in air, we can approximate $n_1$ as unity and let $n$ represent $n_2$ in order to write Eq. 33-49 as

$$\theta_B = \tan^{-1} n \quad \text{(Brewster’s law).}$$  \hspace{1cm} (33-50)

This simplified version of Eq. 33-49 is known as Brewster’s law. Like $\theta_B$, it is named after Sir David Brewster, who found both experimentally in 1812.

**Electromagnetic Waves**

An electromagnetic wave consists of oscillating electric and magnetic fields. The various possible frequencies of electromagnetic waves form a spectrum, a small part of which is visible light. An electromagnetic wave traveling along an $x$ axis has an electric field $\vec{E}$ and a magnetic field $\vec{B}$ with magnitudes that depend on $x$ and $t$:

$$E = E_m \sin(kx - \omega t)$$ and $$B = B_m \sin(kx - \omega t),$$  \hspace{1cm} (33-1, 33-2)

where $E_m$ and $B_m$ are the amplitudes of $\vec{E}$ and $\vec{B}$. The electric field induces the magnetic field and vice versa. The speed of any electromagnetic wave in vacuum is $c$, which can be written as

$$c = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}},$$  \hspace{1cm} (33-5, 33-3)

where $E$ and $B$ are the simultaneous magnitudes of the fields.

**Energy Flow**

The rate per unit area at which energy is transported via an electromagnetic wave is given by the Poynting vector $\vec{S}$:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}.$$  \hspace{1cm} (33-19)

The direction of $\vec{S}$ (and thus of the wave’s travel and the energy transport) is perpendicular to the directions of both $\vec{E}$ and $\vec{B}$. The time-averaged rate per unit area at which energy is transported is $S_{avg}$, which is called the intensity $I$ of the wave:

$$I = \frac{1}{c \mu_0} E_m^2,$$  \hspace{1cm} (33-26)

in which $E_m = E_{rms}/\sqrt{2}$. A point source of electromagnetic waves emits the waves isotropically—that is, with equal intensity in all directions. The intensity of the waves at distance $r$ from a point source of power $P_s$ is

$$I = \frac{P_s}{4\pi r^2},$$  \hspace{1cm} (33-27)

**Radiation Pressure**

When a surface intercepts electromagnetic radiation, a force and a pressure are exerted on the surface. If the radiation is totally absorbed by the surface, the force is

$$F = \frac{1A}{c} \quad \text{(total absorption),}$$  \hspace{1cm} (33-32)

in which $I$ is the intensity of the radiation and $A$ is the area of the surface perpendicular to the path of the radiation. If the radiation is totally reflected back along its original path, the force is

$$F = \frac{2A}{c} \quad \text{(total reflection back along path).}$$  \hspace{1cm} (33-33)

The radiation pressure $p_r$ is the force per unit area:

$$p_r = \frac{I}{c} \quad \text{(total absorption)}$$  \hspace{1cm} (33-34)
and \[ p_r = \frac{2I}{c} \] (total reflection back along path). (33-35)

Polarization  Electromagnetic waves are polarized if their electric field vectors are all in a single plane, called the plane of oscillation. Light waves from common sources are not polarized; that is, they are unpolarized, or polarized randomly.

Polarizing Sheets  When a polarizing sheet is placed in the path of light, only electric field components of the light parallel to the sheet’s polarizing direction are transmitted by the sheet; components perpendicular to the polarizing direction are absorbed. The light that emerges from a polarizing sheet is polarized parallel to the polarizing direction of the sheet.

If the original light is initially unpolarized, the transmitted intensity \[ I \] is half the original intensity \[ I_0 \]:

\[ I = \frac{1}{2} I_0. \] (33-36)

If the original light is initially polarized, the transmitted intensity depends on the angle \[ \theta \] between the polarization direction of the original light and the polarizing direction of the sheet:

\[ I = I_0 \cos^2 \theta. \] (33-38)

Geometrical Optics  Geometrical optics is an approximate treatment of light in which light waves are represented as straight-line rays.

Reflection and Refraction  When a light ray encounters a boundary between two transparent media, a reflected ray and a refracted ray generally appear. Both rays remain in the plane of incidence. The angle of reflection is equal to the angle of incidence, and the angle of refraction is related to the angle of incidence by Snell’s law,

\[ n_2 \sin \theta_2 = n_1 \sin \theta_1 \] (refraction), (33-40)

where \( n_1 \) and \( n_2 \) are the indexes of refraction of the media in which the incident and refracted rays travel.

Total Internal Reflection  A wave encountering a boundary across which the index of refraction decreases will experience total internal reflection if the angle of incidence exceeds a critical angle \( \theta_c \), where

\[ \theta_c = \sin^{-1} \frac{n_2}{n_1} \] (critical angle). (33-45)

Polarization by Reflection  A reflected wave will be fully polarized, with its \( \vec{E} \) vectors perpendicular to the plane of incidence, if it strikes a boundary at the Brewster angle \( \theta_B \), where

\[ \theta_B = \tan^{-1} \frac{n_2}{n_1} \] (Brewster angle). (33-49)

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that rotation, at what angles (measured counterclockwise from the y axis) is the transmission of light through the system eliminated?

7 Figure 33-30 shows rays of monochromatic light passing through three materials a, b, and c. Rank the materials according to the index of refraction, greatest first.

8 Figure 33-31 shows the multiple reflections of a light ray along a glass corridor where the walls are either parallel or perpendicular to one another. If the angle of incidence at point a is 30°, what are the angles of reflection of the light ray at points b, c, d, e, and f?

9 Figure 33-32 shows four long horizontal layers A–D of different materials, with air above and below them. The index of refraction of each material is given. Rays of light are sent into the left end of each layer as shown. In which layer is there the possibility of totally trapping the light in that layer so that, after many reflections, all the light reaches the right end of the layer?

10 The leftmost block in Fig. 33-33 depicts total internal reflection for light inside a material with an index of refraction \( n_1 \) when air is outside the material. A light ray reaching point A from anywhere within the shaded region at the left (such as the ray shown) fully reflects at that point and ends up in the shaded region at the right. The other blocks show similar situations for two other materials. Rank the indexes of refraction of the three materials, greatest first.

11 Each part of Fig. 33-34 shows light that refracts through an interface between two materials. The incident ray (shown gray in the figure) consists of red and blue light. The approximate index of refraction for visible light is indicated for each material. Which of the three parts show physically possible refraction? (Hint: First consider the refraction in general, regardless of the color, and then consider how red and blue light refract differently.)

12 In Fig. 33-35, light travels from material a, through three layers of other materials with surfaces parallel to one another, and then back into another layer of material a. The refractions (but not the associated reflections) at the surfaces are shown. Rank the materials according to index of refraction, greatest first. (Hint: The parallel arrangement of the surfaces allows comparison.)

sec. 33-2 Maxwell’s Rainbow

•1 A certain helium–neon laser emits red light in a narrow band of wavelengths centered at 632.8 nm and with a "wavelength width" (such as on the scale of Fig. 33-1) of 0.0100 nm. What is the corresponding "frequency width" for the emission?

•2 Project Seafarer was an ambitious program to construct an enormous antenna, buried underground on a site about 10 000 km² in area. Its purpose was to transmit signals to submarines while they were deeply submerged. If the effective wavelength were 1.0 \( \times 10^7 \) Earth radii, what would be the (a) frequency and (b) period of the radiations emitted? Ordinarily, electromagnetic radiations do not penetrate very far into conductors such as seawater, and so normal signals cannot reach the submarines.

•3 From Fig. 33-2, approximate the (a) smaller and (b) larger wavelength at which the eye of a standard observer has half the eye’s maximum sensitivity. What are the (c) wavelength, (d) frequency, and (e) period of the light at which the eye is the most sensitive?
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CHAPTER 33 ELECTROMAGNETIC WAVES

4. About how far apart must you hold your hands for them to be separated by 1.0 nano-light-second (the distance light travels in 1.0 ns)?

sec. 33-3 The Traveling Electromagnetic Wave, Qualitatively

5. SSM What inductance must be connected to a 17 pF capacitor in an oscillator capable of generating 550 nm (i.e., visible) electromagnetic waves? Comment on your answer.

6. What is the wavelength of the electromagnetic wave emitted by the oscillator–antenna system of Fig. 33-3 if \( L = 0.253 \mu \text{H} \) and \( C = 25.0 \text{pF} \)?

sec. 33-5 Energy Transport and the Poynting Vector

7. What is the intensity of a traveling plane electromagnetic wave if \( B_{\text{m}} \) is \( 1.0 \times 10^{-4} \text{T} \)?

8. Assume (unrealistically) that a TV station acts as a point source broadcasting isotropically at 1.0 MW. What is the intensity of the transmitted signal reaching Proxima Centauri, the star nearest our solar system, 4.3 ly away? (An alien civilization at that distance might be able to watch X Files. A light-year (ly) is the distance light travels in one year.

9. ILW Some neodymium–glass lasers can provide 100 TW of power in 1.0 ns pulses at a wavelength of 0.26 \( \mu \text{m} \). How much energy is contained in a single pulse?

10. A plane electromagnetic wave has a maximum electric field magnitude of \( 3.20 \times 10^{-6} \text{V/m} \). Find the magnetic field amplitude.

11. ILW A plane electromagnetic wave traveling in the positive direction of an \( x \) axis in vacuum has components \( E_x = 0 \) and \( E_y = (2.0 \text{ V/m}) \cos[(x \times 10^8 \text{ s}^{-1})(t - x/c)] \). (a) What is the amplitude of the magnetic field component? (b) Parallel to which axis does the magnetic field oscillate? (c) When the electric field component is in the positive direction of the \( z \) axis at a certain point \( P \), what is the direction of the magnetic field component there?

12. In a plane radio wave the maximum value of the electric field component is 5.00 V/m. Calculate (a) the maximum value of the magnetic field component and (b) the wave intensity.

13. Sunlight just outside Earth’s atmosphere has an intensity of 1.40 kW/m\(^2\). Calculate (a) \( E_{\text{m}} \) and (b) \( B_{\text{m}} \) for sunlight there, assuming it to be a plane wave.

14. An isotropic point source emits light at wavelength 500 nm, at the rate of 200 W. A light detector is positioned 400 m from the source. What is the maximum rate \( \Delta B/\Delta t \) at which the magnetic component of the light changes with time at the detector’s location?

15. An airplane flying at a distance of 10 km from a radio transmitter receives a signal of intensity 10 \( \mu \text{W/m}^2 \). What is the amplitude of the (a) electric and (b) magnetic component of the signal at the airplane? (c) If the transmitter radiates uniformly over a hemisphere, what is the transmission power?

16. Frank D. Drake, an investigator in the SETI (Search for Extra-Terrestrial Intelligence) program, once said that the large radio telescope in Arecibo, Puerto Rico (Fig. 33-36), “can detect a signal which lays down on the entire surface of the earth a power of only one picowatt.” (a) What is the power that would be received by the Arecibo antenna for such a signal? The antenna diameter is 300 m. (b) What would be the power of an isotropic source at the center of our galaxy that could provide such a signal? The galactic center is \( 2.2 \times 10^4 \) ly away. A light-year is the distance light travels in one year.

** View All Solutions Here **

Fig. 33-36 Problem 16. Radio telescope at Arecibo. (Courtesy Cornell University)

17. The maximum electric field 10 m from an isotropic point source of light is 2.0 V/m. What are (a) the maximum value of the magnetic field and (b) the average intensity of the light there? (c) What is the power of the source?

18. The intensity \( I \) of light from an isotropic point source is determined as a function of distance \( r \) from the source. Figure 33-37 gives intensity \( I \) versus the inverse square \( r^{-2} \) of that distance. The vertical axis scale is set by \( I_s = 200 \text{ W/m}^2 \), and the horizontal axis scale is set by \( r_s^{-2} = 8.0 \text{ m}^2 \). What is the power of the source?

Fig. 33-37 Problem 18.

sec. 33-6 Radiation Pressure

19. SSM High-power lasers are used to compress a plasma (a gas of charged particles) by radiation pressure. A laser generating radiation pulses with peak power \( 1.5 \times 10^7 \text{ MW} \) is focused onto 1.0 mm\(^2\) of high-electron-density plasma. Find the pressure exerted on the plasma if the plasma reflects all the light beams directly back along their paths.

20. Radiation from the Sun reaching Earth (just outside the atmosphere) has an intensity of 1.4 kW/m\(^2\). (a) Assuming that Earth (and its atmosphere) behaves like a flat disk perpendicular to the Sun’s rays and that all the incident energy is absorbed, calculate the force on Earth due to radiation pressure. (b) For comparison, calculate the force due to the Sun’s gravitational attraction.

21. ILW What is the radiation pressure 1.5 m away from a 500 W lightbulb? Assume that the surface on which the pressure is exerted faces the bulb and is perfectly absorbing and that the bulb radiates uniformly in all directions.

22. A black, totally absorbing piece of cardboard of area \( A = 2.0 \text{ cm}^2 \) intercepts light with an intensity of 10 W/m\(^2\) from a camera strobe light. What radiation pressure is produced on the cardboard by the light?

23. Someone plans to float a small, totally absorbing sphere 0.500 m above an isotropic point source of light, so that the upward ra-
solution force from the light matches the downward gravitational force on the sphere. The sphere’s density is 19.0 g/cm³, and its radius is 2.00 mm. (a) What power would be required of the light source? (b) Even if such a source were made, why would the support of the sphere be unstable?

24 SSM It has been proposed that a spaceship might be propelled in the solar system by radiation pressure, using a large sail made of foil. How large must the surface area of the sail be if the radiation force is to be equal in magnitude to the Sun’s gravitational attraction? Assume that the mass of the ship + sail is 1500 kg, that the sail is perfectly reflecting, and that the sail is oriented perpendicular to the Sun’s rays. See Appendix C for needed data. (With a larger sail, the ship is continuously driven away from the Sun.)

25 SSM Prove, for a plane electromagnetic wave that is normally incident on a flat surface, that the radiation pressure on the surface is equal to the energy density in the incident beam. (This relation between pressure and energy density holds no matter what fraction of the incident energy is reflected.)

26 In Fig. 33-38, a laser beam of power 4.60 W and diameter \( D = 2.60 \text{ mm} \) is directed upward at one circular face (of diameter \( d < 2.60 \text{ mm} \)) of a perfectly reflecting cylinder. The cylinder is levitated because the upward radiation force matches the downward gravitational force. If the cylinder’s density is 1.20 g/cm³, what is its height \( H \)?

27 SSM WWW A plane electromagnetic wave, with wavelength 3.0 m, travels in vacuum in the positive direction of an \( x \) axis. The electric field, of amplitude 300 V/m, oscillates parallel to the \( y \) axis. What are the (a) frequency, (b) angular frequency, and (c) angular wave number of the wave? (d) What is the amplitude of the magnetic field component? (e) Parallel to which axis does the magnetic field oscillate? (f) What is the time-averaged rate of energy flow in watts per square meter associated with this wave? The wave uniformly illuminates a surface of area 2.0 m². If the surface totally absorbs the wave, what are (g) the rate at which momentum is transferred to the surface and (h) the radiation pressure on the surface?

28 The average intensity of the solar radiation that strikes normally on a surface just outside Earth’s atmosphere is 1.4 kW/m². (a) What radiation pressure \( p_r \) is exerted on this surface, assuming complete absorption? (b) For comparison, find the ratio of \( p_r \) to Earth’s sea-level atmospheric pressure, which is 1.0 × 10⁵ Pa.

29 SSM A small spaceship with a mass of only 1.5 × 10³ kg (including an astronaut) is drifting in outer space with negligible gravitational forces acting on it. If the astronaut turns on a 10 kW laser beam, what speed will the ship attain in 1.0 day because of the momentum carried away by the beam?

30 A small laser emits light at power 5.00 mW and wavelength 633 nm. The laser beam is focused (narrowed) until its diameter matches the 1266 nm diameter of a sphere placed in its path. The sphere is perfectly absorbing and has density 5.00 × 10³ kg/m³. What are (a) the beam intensity at the sphere’s location, (b) the radiation pressure on the sphere, (c) the magnitude of the corresponding force, and (d) the magnitude of the acceleration that force alone would give the sphere?

31 As a comet swings around the Sun, ice on the comet’s surface vaporizes, releasing trapped dust particles and ions. The ions, because they are electrically charged, are forced by the electrically charged solar wind into a straight ion tail that points radially away from the Sun (Fig. 33-39). The (electrically neutral) dust particles are pushed radially outward from the Sun by the radiation force on them from sunlight. Assume that the dust particles are spherical, have density 3.5 × 10³ kg/m³, and are totally absorbing. (a) What radius must a particle have in order to follow a straight path, like path 2 in the figure? (b) If its radius is larger, does its path curve away from the Sun (like path 1) or toward the Sun (like path 3)?

sec. 33.7 Polarization

32 In Fig. 33-40, initially unpolarized light is sent into a system of three polarizing sheets whose polarization directions make angles of \( \theta_1 = \theta_3 = 50^\circ \) with the direction of the \( y \) axis. What percentage of the initial intensity is transmitted by the system? (Hint: Be careful with the angles.)

33 SSM In Fig. 33-40, initially unpolarized light is sent into a system of three polarizing sheets whose polarization directions make angles of \( \theta_1 = 40^\circ \), \( \theta_2 = 20^\circ \), and \( \theta_3 = 40^\circ \) with the direction of the \( y \) axis. What percentage of the light’s initial intensity is transmitted by the system? (Hint: Be careful with the angles.)

34 SSM In Fig. 33-41, a beam of unpolarized light, with intensity 43 W/m², is sent into a system of two polarizing sheets with polarization directions at angles \( \theta_1 = 70^\circ \) and \( \theta_2 = 90^\circ \) to the \( y \) axis. What is the intensity of the light transmitted by the system?

35 ILW In Fig. 33-41, a beam of light, with intensity 43 W/m² and polarization parallel to the \( y \) axis, is sent into a system of two polarizing sheets with polarization directions at angles \( \theta_1 = 70^\circ \) and \( \theta_2 = 90^\circ \) to the \( y \) axis. What is the intensity of the light transmitted by the two-sheet system?

36 At a beach the light is generally partially polarized due to reflections off sand and water. At a particular beach on a particular day near sundown, the horizontal component of the electric field vector is 2.3 times the vertical component. A standing sunbather puts on polarizing sunglasses; the glasses eliminate the
horizontal field component. (a) What fraction of the light intensity received before the glasses were put on now reaches the sunbather’s eyes? (b) The sunbather, still wearing the glasses, lies on his side. What fraction of the light intensity received before the glasses were put on now reaches his eyes?

**37** SSM WWW We want to rotate the direction of polarization of a beam of polarized light through 90° by sending the beam through one or more polarizing sheets. (a) What is the minimum number of sheets required? (b) What is the minimum number of sheets required if the transmitted intensity is to be more than 60% of the original intensity?

**38** In Fig. 33-42, unpolarized light is sent into a system of three polarizing sheets. The angles \( \theta_1, \theta_2, \) and \( \theta_3 \) of the polarizing directions are measured counterclockwise from the positive direction of the \( y \) axis (they are not drawn to scale). Angles \( \theta_1 \) and \( \theta_2 \) are fixed, but angle \( \theta_3 \) can be varied. Figure 33-43 gives the intensity of the light emerging from sheet 3 as a function of \( \theta_3 \). (The scale of the intensity axis is not indicated.) What percentage of the light’s initial intensity is transmitted by the three-sheet system when \( \theta_2 = 30° \)?

**39** Unpolarized light of intensity 10 mW/m² is sent into a polarizing sheet as in Fig. 33-11. What are (a) the amplitude of the electric field component of the transmitted light and (b) the radiation pressure on the sheet due to its absorbing some of the light?

**40** In Fig. 33-42, unpolarized light is sent into a system of three polarizing sheets. The angles \( \theta_1, \theta_2, \) and \( \theta_3 \) of the polarizing directions are measured counterclockwise from the positive direction of the \( y \) axis (they are not drawn to scale). Angles \( \theta_1 \) and \( \theta_2 \) are fixed, but angle \( \theta_3 \) can be varied. Figure 33-44 gives the intensity of the light emerging from sheet 3 as a function of \( \theta_3 \). (The scale of the intensity axis is not indicated.) What percentage of the light’s initial intensity is transmitted by the three-sheet system when \( \theta_2 = 90° \)?

**41** A beam of polarized light is sent into a system of two polarizing sheets. Relative to the polarization direction of that incident light, the polarizing directions of the sheets are at angles \( \theta \) for the first sheet and 90° for the second sheet. If 0.10 of the incident intensity is transmitted by the two sheets, what is \( \theta \)?

**42** In Fig. 33-41, unpolarized light is sent into a system of two polarizing sheets. The angles \( \theta_1 \) and \( \theta_2 \) of the polarizing directions of the sheets are measured counterclockwise from the positive direction of the \( y \) axis (they are not drawn to scale). Angle \( \theta_1 \) is fixed but angle \( \theta_2 \) can be varied. Figure 33-45 gives the intensity of the light emerging from sheet 2 as a function of \( \theta_2 \). (The scale of the intensity axis is not indicated.) What percentage of the light’s initial intensity is transmitted by the two-sheet system when \( \theta_2 = 90° \)?

**43** A beam of partially polarized light can be considered to be a mixture of polarized and unpolarized light. Suppose we send such a beam through a polarizing filter and then rotate the filter through 360° while keeping it perpendicular to the beam. If the transmitted intensity varies by a factor of 5.0 during the rotation, what fraction of the intensity of the original beam is associated with the beam’s polarized light?

**44** In Fig. 33-42, unpolarized light is sent into a system of three polarizing sheets, which transmits 0.0500 of the initial light intensity. The polarizing directions of the first and third sheets are at angles \( \theta_1 = 0° \) and \( \theta_3 = 90° \). What are the (a) smaller and (b) larger possible values of angle \( \theta_2 \) (less than 90°) for the polarizing direction of sheet 2?

**33.8 Reflection and Refraction**

**45** When the rectangular metal tank in Fig. 33-46 is filled to the top with an unknown liquid, observer \( O \), with eyes level with the top of the tank, can just see corner \( E \). A ray that refracts toward \( O \) at the top surface of the liquid is shown. If \( D = 85.0 \text{ cm} \) and \( L = 1.10 \text{ m} \), what is the index of refraction of the liquid?

**46** In Fig. 33-47a, a light ray in an underlying material is incident at angle \( \theta_i \) on a boundary with water, and some of the light refracts into the water. There are two choices of underlying material. For each, the angle of refraction \( \theta_r \) versus the incident angle \( \theta_i \) is given in Fig. 33-47b. The horizontal axis scale is set by \( \theta_i = 90° \). Without calculation, determine whether the index of refraction of (a) material 1 and (b) material 2 is greater or less than 1.
than the index of water ($n = 1.33$). What is the index of refraction of (c) material 1 and (d) material 2?

![Fig. 33-47](image)

**Problem 46.**

**47** Light in vacuum is incident on the surface of a glass slab. In the vacuum the beam makes an angle of $32.0°$ with the normal to the surface, while in the glass it makes an angle of $21.0°$ with the normal. What is the index of refraction of the glass?

**48** In Fig. 33-48a, a light ray in water is incident at angle $\theta_1$ on a boundary with an underlying material, into which some of the light refracts. There are two choices of underlying material. For each, the angle of refraction $\theta_2$ versus the incident angle $\theta_1$ is given in Fig. 33-48b. The vertical axis scale is set by $\theta_2 = 90°$. Without calculation, determine whether the index of refraction of (a) material 1 and (b) material 2 is greater or less than the index of water ($n = 1.33$). What is the index of refraction of (c) material 1 and (d) material 2?

![Fig. 33-48](image)

**Problem 48.**

**49** Figure 33-49 shows light reflecting from two perpendicular reflecting surfaces $A$ and $B$. Find the angle between the incoming ray $i$ and the outgoing ray $r'$.

![Fig. 33-49](image)

**Problem 49.**

**50** In Fig. 33-50a, a beam of light in material 1 is incident on a boundary at an angle $\theta_1 = 40°$. Some of the light travels through material 2, and then some of it emerges into material 3. The two boundaries between the three materials are parallel. The final direction of the beam depends, in part, on the index of refraction $n_3$ of the third material. Figure 33-50b gives the angle of refraction $\theta_3$ in that material versus $n_3$ for a range of possible $n_3$ values. The vertical axis scale is set by $\theta_3 = 30.0°$ and $\theta_3 = 50.0°$. (a) What is the index of refraction of material 1, or is the index impossible to calculate without more information? (b) What is the index of refraction of material 2, or is the index impossible to calculate without more information? (c) If $\theta_1$ is changed to $70°$ and the index of refraction of material 3 is 2.4, what is $\theta_3$?

![Fig. 33-50](image)

**Problem 50.**

**51** In Fig. 33-51, light is incident at angle $\theta_1 = 40.1°$ on a boundary between two transparent materials. Some of the light travels down through the next three layers of transparent materials, while some of it reflects upward and then escapes into the air. If $n_1 = 1.30$, $n_2 = 1.40$, $n_3 = 1.52$, and $n_4 = 1.45$, what is the value of (a) $\theta_3$ in the air and (b) $\theta_5$ in the bottom material?

![Fig. 33-51](image)

**Problem 51.**

**52** In Fig. 33-52a, a beam of light in material 1 is incident on a boundary at an angle of $\theta_1 = 30°$. The extent of refraction of the light into material 2 depends, in part, on the index of refraction $n_2$ of material 2. Figure 33-52b gives the angle of refraction $\theta_2$ in that material versus $n_2$ for a range of possible $n_2$ values. The vertical axis scale is set by $\theta_2 = 20.0°$ and $\theta_2 = 40.0°$. (a) What is the index of refraction of material 1? (b) If the incident angle is changed to $60°$ and material 2 has $n_2 = 2.4$, then what is angle $\theta_2$?

![Fig. 33-52](image)

**Problem 52.**

**View All Solutions Here**
In Fig. 33-53, a ray is incident on one face of a triangular glass prism in air. The angle of incidence $\theta$ is chosen so that the emerging ray also makes the same angle $\theta$ with the normal to the other face. Show that the index of refraction $n$ of the glass prism is given by

$$n = \frac{\sin \frac{\phi}{2}}{\sin \frac{\psi}{2}},$$

where $\phi$ is the vertex angle of the prism and $\psi$ is the deviation angle, the total angle through which the beam is turned in passing through the prism. (Under these conditions the deviation angle $\psi$ has the smallest possible value, which is called the angle of minimum deviation.)

In Fig. 33-54, a beam of white light is incident at angle $\theta = 50^\circ$ on a common window pane (shown in cross section). For the pane’s type of glass, the index of refraction for visible light ranges from 1.524 at the blue end of the spectrum to 1.609 at the red end. The two sides of the pane are parallel. What is the angular spread of the colors in the beam (a) when the light enters the pane and (b) when it emerges from the opposite side? (Hint: When you look at an object through a window pane, are the colors in the light from the object dispersed as shown in, say, Fig. 33-20?)

In Fig. 33-55, a 2.00-m-long vertical pole extends from the bottom of a swimming pool to a point 50.0 cm above the water. Sunlight is incident at angle $\theta = 55.0^\circ$. What is the length of the shadow of the pole on the level bottom of the pool?

Rainbows from square drops. Suppose that, on some surreal world, raindrops had a square cross section and always fell with one face horizontal. Figure 33-56 shows such a falling drop, with a white beam of sunlight incident at $\theta = 70.0^\circ$ at point $P$. The part of the light that enters the drop then travels to point $A$, where some of it refracts out into the air and the rest reflects. That reflected light then travels to point $B$, where again some of the light refracts out into the air and the rest reflects. What is the difference in the angles of the red light ($n = 1.331$) and the blue light ($n = 1.343$) that emerge at (a) point $A$ and (b) point $B$? (This angular difference in the light emerging at, say, point $A$ would be the rainbow’s angular width.)

A point source of light is 80.0 cm below the surface of a body of water. Find the diameter of the circle at the surface through which light emerges from the water.

The index of refraction of benzene is 1.8. What is the critical angle for a light ray traveling in benzene toward a flat layer of air above the benzene?

In Fig. 33-57, a ray of light is perpendicular to the face $ab$ of a glass prism ($n = 1.52$). Find the largest value for the angle $\phi$ so that the ray is totally reflected at face $ac$ if the prism is immersed (a) in air and (b) in water.

In Fig. 33-58, light from ray $A$ refracts from material 1 ($n_1 = 1.60$) into a thin layer of material 2 ($n_2 = 1.80$), crosses that layer, and is then incident at the critical angle on the interface between materials 2 and 3 ($n_3 = 1.30$). (a) What is the value of the index of refraction $n_3$? (b) What is the value of incidence angle $\theta_3$? (c) What is the value of reflection angle $\theta_3'$? (d) If $\theta_3$ is decreased, does part of the light refract into material 3?

Light from ray $B$ refracts from material 1 into the thin layer, crosses that layer, and is then incident at the critical angle on the interface between materials 2 and 3. (a) What is the value of the index of refraction $n_2$? (b) What is the value of incidence angle $\theta_2$? (c) What is the value of reflection angle $\theta_2'$? (d) If $\theta_2$ is decreased, does part of the light refract into material 3?

In Fig. 33-59, light initially in material 1 refracts into material 2, crosses that material, and is then incident at the critical angle on the interface between materials 2 and 3. The indexes of refraction are $n_1 = 1.60$, $n_2 = 1.40$, and $n_3 = 1.20$. (a) What is angle $\theta$? (b) If $\theta$ is increased, is there refraction of light into material 3?

A catfish is 2.00 m below the surface of a smooth lake. (a) What is the diameter of the circle on the surface through which the fish can see the world outside the water? (b) If the fish descends, does the diameter of the circle increase, decrease, or remain the same?

In Fig. 33-60, light enters a 90° triangular prism at point $P$ with incidence angle $\theta$, and then some of it refracts at point $Q$ with an angle of refraction of 90°. (a) What is the index of refraction of the prism in terms of $\theta$? (b) What, numerically, is the maximum value that the index of refraction can have? Does light emerge at $Q$ if the incidence angle at $P$ is (c) increased slightly and (d) decreased slightly?
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75 SSM In Fig. 33-65, a light ray enters a glass slab at point A at incident angle \( \theta_i = 45.0^\circ \) and then undergoes total internal reflection at point B. What minimum value for the index of refraction of the glass can be inferred from this information?

76 In Fig. 33-66, unpolarized light with an intensity of 25 W/m\(^2\) is sent into a system of four polarizing sheets with polarizing directions at angles \( \theta_1 = 40^\circ \), \( \theta_2 = 20^\circ \), \( \theta_3 = 20^\circ \), and \( \theta_4 = 30^\circ \). What is the intensity of the light that emerges from the system?

77 Rainbow. Figure 33-67 shows a light ray entering and then leaving a falling, spherical raindrop after one internal reflection (see Fig. 33-21a). The final direction of travel is deviated (turned) from the initial direction of travel by angular deviation \( \theta_{dev} \). (a) Show that \( \theta_{dev} \) is

\[
\theta_{dev} = 180^\circ + 2\theta_i - 4\theta_r,
\]

where \( \theta_i \) is the angle of incidence of the ray on the drop and \( \theta_r \) is the angle of refraction of the ray within the drop. (b) Using Snell’s law, substitute for \( \theta_i \) in terms of \( \theta_r \) and the index of refraction \( n \) of the water. Then, on a graphing calculator or with a computer graphing package, graph \( \theta_{dev} \) versus \( \theta_r \) for the range of possible \( \theta_r \) values and for \( n = 1.331 \) for red light and \( n = 1.333 \) for blue light.

The red-light curve and the blue-light curve have different minima, which means that there is a different angle of minimum deviation for each color. The light of any given color that leaves the drop at that color’s angle of minimum deviation is especially bright because rays bunch up at that angle. Thus, the bright red light leaves the drop at one angle and the bright blue light leaves it at another angle.

Determine the angle of minimum deviation from the \( \theta_{dev} \) curve for (c) red light and (d) blue light. (e) If these colors form the inner and outer edges of a rainbow (Fig. 33-21a), what is the angular width of the rainbow?

78 The primary rainbow described in Problem 77 is the type commonly seen in regions where rainbows appear. It is produced by light reflecting once inside the drops. Rarer is the secondary rainbow described in Section 33-8, produced by light reflecting twice inside the drops (Fig. 33-68a). (a) Show that the angular deviation of light entering and then leaving a spherical water drop is

\[
\theta_{dev} = (180^\circ)k + 2\theta_i - 2(k + 1)\theta_r,
\]

where \( k \) is the number of internal reflections. Using the procedure of Problem 77, find the angle of minimum deviation for (b) red light and (c) blue light in a secondary rainbow. (d) What is the angular width of that rainbow (Fig. 33-21d)?

The tertiary rainbow depends on three internal reflections (Fig. 33-68b). It probably occurs but, as noted in Section 33-8, cannot be seen because it is very faint and lies in the bright sky surrounding the Sun. What is the angle of minimum deviation for (e) the red light and (f) the blue light in this rainbow? (g) What is the rainbow’s angular width?

79 SSM (a) Prove that a ray of light incident on the surface of a sheet of plate glass of thickness \( t \) emerges from the opposite face parallel to its initial direction but displaced sideways, as in Fig. 33-69. (b) Show that, for small angles of incidence \( \theta \), this displacement is given by

\[
x = \frac{t(n - 1)}{n},
\]

where \( n \) is the index of refraction of the glass and \( \theta \) is measured in radians.

80 An electromagnetic wave is traveling in the negative direction of a \( y \) axis. At a particular position and time, the electric field is directed along the positive direction of the \( z \) axis and has a magnitude of 100 V/m. What are the (a) magnitude and (b) direction of the corresponding magnetic field?
81 The magnetic component of a polarized wave of light is 
\[ B_y = (4.0 \times 10^{-6}) \sin((1.57 \times 10^7 m^{-1})y + \omega t). \]
(a) Parallel to which axis is the light polarized? What are the (b) frequency and (c) intensity of the light?

82 In Fig. 33-70, unpolarized light is sent into the system of three polarizing sheets, where the polarization directions of the first and third sheets are at angles \( \theta_1 = 30^\circ \) (counterclockwise) and \( \theta_2 = 30^\circ \) (clockwise). What fraction of the initial light intensity emerges from the system?

83 **SSM** A ray of white light traveling through fused quartz is incident at a quartz–air interface at angle \( \theta_i \). Assume that the index of refraction of quartz is \( n = 1.456 \) at the red end of the visible range and \( n = 1.470 \) at the blue end. If \( \theta_i \) is (a) 42.00°, (b) 43.10°, and (c) 44.00°, is the refracted light white, white dominated by the blue end of the visible range, or white dominated by the blue end of the visible range, or is there no refracted light?

84 Three polarizing sheets are crossed; the first and third are crossed; the one between has its polarization direction at 45.0° to the polarizing directions of the other two. What fraction of the intensity of an originally unpolarized beam is transmitted by the stack?

85 In a region of space where gravitational forces can be neglected, a sphere is accelerated by a uniform light beam of intensity 6.0 mW/m². The sphere is totally absorbing and has a radius of 2.0 \( \mu \)m and a uniform density of \( 5.0 \times 10^3 \) kg/m³. What is the magnitude of the sphere’s acceleration due to the light?

86 An unpolarized beam of light is sent into a stack of four polarizing sheets, oriented so that the angle between the polarization directions of adjacent sheets is 30°. What fraction of the incident intensity is transmitted by the system?

87 **SSM** During a test, a NATO surveillance radar system, operating at 12 GHz at 180 kW of power, attempts to detect an incoming stealth aircraft at 90 km. Assume that the radar beam is emitted uniformly over a hemisphere. (a) What is the intensity of the beam when the beam reaches the aircraft’s location? The aircraft reflects radar waves as though it has a cross-sectional area of only 0.22 m². (b) What is the power of the aircraft’s reflection? Assume that the beam is reflected uniformly over a hemisphere. Back at the radar site, what are (c) the intensity, (d) the maximum value of the electric field vector, and (e) the rms value of the magnetic field of the reflected radar beam?

88 The magnetic component of an electromagnetic wave in vacuum has an amplitude of 85.8 nT and an angular wave number of 4.00 m⁻¹. What are (a) the frequency of the wave, (b) the rms value of the electric component, and (c) the intensity of the light?

89 Calculate the (a) upper and (b) lower limit of the Brewster angle for white light incident on fused quartz. Assume that the wavelength limits of the light are 400 and 700 nm.

90 In Fig. 33-71, two light rays pass from air through five layers of transparent plastic and then back into air. The layers have parallel interfaces and unknown thicknesses; their indexes of refraction are \( n_1 = 1.7, n_2 = 1.6, n_3 = 1.5, n_4 = 1.4, \) and \( n_5 = 1.6 \). Ray \( b \) is incident at angle \( \theta_i = 20^\circ \). Relative to a normal at the last interface, at what angle do (a) ray \( a \) and (b) ray \( b \) emerge? (Hint: Solving the problem algebraically can save time.) If the air at the left and right sides in the figure were, instead, glass with index of refraction 1.5, at what angle would (c) ray \( a \) and (d) ray \( b \) emerge?

91 A helium–neon laser, radiating at 632.8 nm, has a power output of 3.6 mW. The beam diverges (spreads) at angle \( \theta = 0.17 \) mrad (Fig. 33-72). (a) What is the intensity of the beam 40 m from the laser? (b) What is the power of a point source providing that intensity at that distance?

92 In about A.D. 150, Claudius Ptolemy gave the following measured values for the angle of incidence \( \theta_i \) and the angle of refraction \( \theta_r \) for a light beam passing from air to water:

<table>
<thead>
<tr>
<th>( \theta_i )</th>
<th>( \theta_r )</th>
<th>( \theta_i )</th>
<th>( \theta_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>8°</td>
<td>50°</td>
<td>35°</td>
</tr>
<tr>
<td>20°</td>
<td>15°30′</td>
<td>60°</td>
<td>40°30′</td>
</tr>
<tr>
<td>30°</td>
<td>22°30′</td>
<td>70°</td>
<td>45°30′</td>
</tr>
<tr>
<td>40°</td>
<td>29°</td>
<td>80°</td>
<td>50°</td>
</tr>
</tbody>
</table>

Assuming these data are consistent with the law of refraction, use them to find the index of refraction of water. These data are interesting as perhaps the oldest recorded physical measurements.

93 A beam of initially unpolarized light is sent through two polarizing sheets placed one on top of the other. What must be the angle between the polarizing directions of the sheets if the intensity of the transmitted light is to be one-third the incident intensity?