Light Quanta—Photons
An electromagnetic wave (light) is quantized, and its quanta are called photons. For a light wave of frequency \( f \) and wavelength \( \lambda \), the energy \( E \) and momentum magnitude \( p \) of a photon are

\[
E = hf \quad \text{(photon energy)} \quad (38-2)
\]

and

\[
p = \frac{hf}{c} = \frac{h}{\lambda} \quad \text{(photon momentum).} \quad (38-7)
\]

Photoelectric Effect
When light of high enough frequency falls on a clean metal surface, electrons are emitted from the surface by photon–electron interactions within the metal. The governing relation is

\[
hf = K_{\text{max}} + \Phi, \quad (38-5)
\]

in which \( hf \) is the photon energy, \( K_{\text{max}} \) is the kinetic energy of the most energetic emitted electrons, and \( \Phi \) is the work function of the target material—that is, the minimum energy an electron must have if it is to emerge from the surface of the target. If \( hf \) is less than \( \Phi \), electrons are not emitted.

Compton Shift
When x rays are scattered by loosely bound electrons in a target, some of the scattered x rays have a longer wavelength than do the incident x rays. This **Compton shift** (in wavelength) is given by

\[
\Delta \lambda = \frac{h}{m c} \left(1 - \cos \phi \right), \quad (38-11)
\]

in which \( \phi \) is the angle at which the x rays are scattered.

Light Waves and Photons
When light interacts with matter, energy and momentum are transferred via photons. When light is in transit, however, we interpret the light wave as a **probability wave**, in which the probability (per unit time) that a photon can be detected is proportional to \( E_x \), where \( E_x \) is the amplitude of the oscillating electric field of the light wave at the detector.

Matter Waves
A moving particle such as an electron or a proton can be described as a **matter wave**; its wavelength (called the de Broglie wavelength) is given by \( \lambda = \hbar/p \), where \( p \) is the magnitude of the particle’s momentum.

**The Wave Function**
A matter wave is described by its wave function \( \Psi(x, y, z, t) \), which can be separated into a space-dependent part \( \phi(x, y, z) \) and a time-dependent part \( e^{-i\omega t} \). For a particle of mass \( m \) moving in the \( x \) direction with constant total energy \( E \) through a region in which its potential energy is \( U(x) \), the phase \( \phi(x) \) can be found by solving the simplified Schrödinger equation:

\[
\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{\hbar^2} [E - U(x)]\psi = 0. \quad (38-15)
\]

A matter wave, like a light wave, is a probability wave in the sense that if a particle detector is inserted into the wave, the probability that the detector will register a particle during any specified time interval is proportional to \( |\psi|^2 \), a quantity called the **probability density**.

For a free particle—that is, a particle for which \( U(x) = 0 \)—moving in the \( x \) direction, \( |\psi|^2 \) has a constant value for all positions along the \( x \) axis.

**Heisenberg’s Uncertainty Principle**
The probabilistic nature of quantum physics places an important limitation on detecting a particle’s position and momentum. That is, it is not possible to measure the position \( \vec{r} \) and the momentum \( \vec{p} \) of a particle simultaneously with unlimited precision. The uncertainties in the components of these quantities are given by

\[
\begin{align*}
\Delta x \cdot \Delta p_x &\geq \hbar \\
\Delta y \cdot \Delta p_y &\geq \hbar \\
\Delta z \cdot \Delta p_z &\geq \hbar.
\end{align*}
\]

**Barrier Tunneling**
According to classical physics, an incident particle will be reflected from a potential energy barrier whose height is greater than the particle’s kinetic energy. According to quantum physics, however, the particle has a finite probability of tunneling through such a barrier. The probability that a given particle of mass \( m \) and energy \( E \) will tunnel through a barrier of height \( U_b \) and thickness \( L \) is given by the transmission coefficient \( T \):

\[
T = e^{-2b}, \quad (38-21)
\]

where

\[
b = \sqrt{\frac{8\pi^2m(U_b - E)}{\hbar^2}}. \quad (38-22)
\]

**Questions**

1. Photon \( A \) has twice the energy of photon \( B \). (a) Is the momentum of \( A \) less than, equal to, or greater than that of \( B \)? (b) Is the wavelength of \( A \) less than, equal to, or greater than that of \( B \)?

2. In the photoelectric effect (for a given target and a given frequency of the incident light), which of these quantities, if any, depend on the intensity of the incident light beam: (a) the maximum kinetic energy of the electrons, (b) the maximum photoelectric current, (c) the stopping potential, (d) the cutoff frequency?

3. According to the figure for Checkpoint 2, is the maximum kinetic energy of the ejected electrons greater for a target made of sodium or of potassium for a given frequency of incident light?

4. Photoelectric effect: Figure 38-18 gives the stopping voltage \( V \) versus the wavelength \( \lambda \) of light for three different materials. Rank the materials according to their work function, greatest first.

5. A metal plate is illuminated with light of a certain frequency. Which of the following determine whether or not electrons are ejected: (a) the intensity of the light, (b) how long the plate is exposed to the light, (c) the thermal conductivity of the plate, (d) the area of the plate, (e) the material of which the plate is made?

6. Let \( K \) be the kinetic energy that a stationary free electron gains...
when a photon scatters from it. We can plot $K$ versus the angle $\phi$ at which the photon scatters; see curve 1 in Fig. 38-19. If we switch the target to be a stationary free proton, does the end point of the graph shift (a) upward as suggested by curve 2, (b) downward as suggested by curve 3, or (c) remain the same?

7 In a Compton-shift experiment, light (in the x-ray range) is scattered in the forward direction, at $\phi = 0$ in Fig. 38-3. What fraction of the light’s energy does the electron acquire?

8 Compton scattering. Figure 38-20 gives the Compton shift $\Delta \lambda$ versus scattering angle $\phi$ for three different stationary target particles. Rank the particles according to their mass, greatest first.

9 (a) If you double the kinetic energy of a nonrelativistic particle, how does its de Broglie wavelength change? (b) What if you double the speed of the particle?

10 Figure 38-21 shows an electron moving (a) opposite an electric field, (b) in the same direction as an electric field, (c) in the same direction as a magnetic field, (d) perpendicular to a magnetic field. For each situation, is the de Broglie wavelength of the electron increasing, decreasing, or remaining the same?

11 At the left in Fig. 38-16, why are the minima nonzero?

12 An electron and a proton have the same kinetic energy. Which has the greater de Broglie wavelength?

13 The following nonrelativistic particles all have the same kinetic energy. Rank them in order of their de Broglie wavelengths, greatest first: electron, alpha particle, neutron.

14 Figure 38-22 shows an electron moving through several regions where uniform electric potentials $V$ have been set up. Rank the three regions according to the de Broglie wavelength of the electron there, greatest first.

15 The table gives relative values for three situations for the barrier tunneling experiment of Figs. 38-14 and 38-15. Rank the situations according to the probability of the electron tunneling through the barrier, greatest first.

<table>
<thead>
<tr>
<th>Electron Energy</th>
<th>Barrier Height</th>
<th>Barrier Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$5E$</td>
<td>$L$</td>
</tr>
<tr>
<td>$E$</td>
<td>$17E$</td>
<td>$L/2$</td>
</tr>
<tr>
<td>$E$</td>
<td>$2E$</td>
<td>$2L$</td>
</tr>
</tbody>
</table>

16 For three experiments, Fig. 38-23 gives the transmission coefficient $T$ for electron tunneling through a potential barrier, plotted versus barrier thickness $L$. The de Broglie wavelengths of the electrons are identical in the three experiments. The only difference in the physical setups is the barrier heights $U_b$. Rank the three experiments according to $U_b$, greatest first.

** View All Solutions Here **

sec. 38-2 The Photon, the Quantum of Light

1 Monochromatic light (that is, light of a single wavelength) is to be absorbed by a sheet of photographic film and thus recorded on the film. Photon absorption will occur if the photon energy equals or exceeds 0.6 eV, the smallest amount of energy needed to dissociate an AgBr molecule in the film. (a) What is the greatest wavelength of light that can be recorded by the film? (b) In what region of the electromagnetic spectrum is this wavelength located?

2 How fast must an electron move to have a kinetic energy equal to the photon energy of sodium light at wavelength 590 nm?

3 At what rate does the Sun emit photons? For simplicity, assume that the Sun’s entire emission at the rate of $3.9 \times 10^{26}$ W is at the single wavelength of 550 nm.

4 A helium–neon laser emits red light at wavelength $\lambda = 633$ nm in a beam of diameter $3.5 \text{ mm}$ and at an energy-emission rate of $5.0 \text{ mW}$. A detector in the beam’s path totally absorbs the beam. At what rate per unit area does the detector absorb photons?

5 The meter was once defined as 1 650 763.73 wavelengths of the orange light emitted by a source containing krypton-86 atoms. What is the photon energy of that light?
The yellow-colored light from a highway sodium lamp is brightest at a wavelength of 589 nm. What is the photon energy for light at that wavelength?

A light detector (your eye) has an area of $2.00 \times 10^{-6} \, \text{m}^2$ and absorbs 80% of the incident light, which is at wavelength 500 nm. The detector faces an isotropic source, 3.00 m from the source. If the detector absorbs photons at the rate of exactly 1.00 photon/cm$^2$ s, at what rate are photons absorbed by the screen in the central disk of the diffraction pattern?

The beam emerging from a 1.5 W argon laser ($\lambda = 515 \, \text{nm}$) has a diameter $d$ of 3.0 mm. The beam is focused by a lens system with an effective focal length $f_0$ of 2.5 mm. The focused beam strikes a totally absorbing screen, where it forms a circular diffraction pattern whose central disk has a radius $R$ given by $1.22 \frac{f_0}{d}$. It can be shown that 84% of the incident energy ends up within this central disk. At what rate are photons absorbed by the screen in the central disk of the diffraction pattern?

A 100 W sodium lamp ($\lambda = 589 \, \text{nm}$) radiates energy uniformly in all directions. (a) At what rate are photons emitted by the lamp? (b) At what distance from the lamp will a totally absorbing screen absorb photons at the rate of 1.00 photon/cm$^2$ s? (c) What is the photon flux (photons per unit area per unit time) on a small screen 2.00 m from the lamp?

A satellite in Earth orbit maintains a panel of solar cells of area 2.60 m$^2$ perpendicular to the direction of the Sun’s light rays. The intensity of the light at the panel is 1.39 kW/m$^2$. (a) At what rate does solar energy arrive at the panel? (b) At what rate are solar photons absorbed by the panel? Assume that the solar radiation is monochromatic, with a wavelength of 550 nm, and that all the solar radiation striking the panel is absorbed. (c) How long would it take for a “mole of photons” to be absorbed by the panel?

A satellite coated with platinum, a metal with a very large work function 1.80 eV, is designed to minimize such charging because it can ruin the sensitive microelectronics. Suppose a satellite is orbiting satellite can become charged by the photoelectric effect with visible light. Which of the following are suitable (work functions are in parentheses): tantalum (4.2 eV), tungsten (4.5 eV), aluminum (4.2 eV), barium (2.5 eV), lithium (2.3 eV)?

(a) If the work function for a certain metal is 1.8 eV, what is the stopping potential for electrons ejected from the metal when light of wavelength 400 nm shines on the metal? (b) What is the maximum speed of the ejected electrons?

Suppose the fractional efficiency of a cesium surface (with work function 1.80 eV) is 0.10 $\times 10^{-16}$ that is, on average one electron is ejected for every 10$^{16}$ photons that reach the surface. What would be the current of electrons ejected from such a surface if it were illuminated with 600 nm light from a 2.00 mW laser and all the ejected electrons took part in the charge flow?

X rays with a wavelength of 71 pm are directed onto a gold foil and eject tightly bound electrons from the gold atoms. The ejected electrons then move in circular paths of radius $r$ in a region of uniform magnetic field $B$. For the fastest of the ejected electrons, the product $Br$ is equal to $1.88 \times 10^{-4} \, \text{T} \cdot \text{m}$. Find (a) the maximum kinetic energy of those electrons and (b) the work done in removing them from the gold atoms.

The wavelength associated with the cutoff frequency for silver is 325 nm. Find the maximum kinetic energy of electrons ejected from a silver surface by ultraviolet light of wavelength 254 nm.

Light of wavelength 200 nm shines on an aluminum surface; 4.20 eV is required to eject an electron. What is the kinetic energy of (a) the fastest and (b) the slowest ejected electrons? (c) What is the stopping potential for this situation? (d) What is the cutoff wavelength for aluminum?

In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300 nm and a stopping potential of 0.820 V for a wavelength of 400 nm. From these data find (a) a value for the Planck constant, (b) the work function $\Phi$ for sodium, and (c) the cutoff wavelength $\lambda_0$ for sodium.

The stopping potential for electrons emitted from a surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. (a) What is this new wavelength? (b) What is the work function for the surface?

An orbiting satellite can become charged by the photoelectric effect when sunlight ejects electrons from its outer surface. Satellites must be designed to minimize such charging because it can ruin the sensitive microelectronics. Suppose a satellite is coated with platinum, a metal with a very large work function ($\Phi = 5.32 \, \text{eV}$). Find the longest wavelength of incident sunlight that can eject an electron from the platinum.
sec. 38-4 Photons Have Momentum

•27 SSM Light of wavelength 2.40 μm is directed onto a target containing free electrons. (a) Find the wavelength of light scattered at 30.0° from the incident direction. (b) Do the same for a scattering angle of 120°.

•28 (a) In MeV/c, what is the magnitude of the momentum associated with a photon having an energy equal to the electron rest energy? What are the (b) wavelength and (c) frequency of the corresponding radiation?

•29 What (a) frequency, (b) photon energy, and (c) photon momentum magnitude (in keV/c) are associated with x rays having wavelength 35.0 pm?

•30 What is the maximum wavelength shift for a Compton collision between a photon and a free proton?

•31 What percentage increase in wavelength leads to a 75% loss of photon energy in a photon–free electron collision?

•32 X rays of wavelength 0.0100 nm are directed in the positive direction of an x axis onto a target containing loosely bound electrons. For Compton scattering from one of those electrons, at an angle of 180°, what are (a) the Compton shift, (b) the corresponding change in photon energy, (c) the kinetic energy of the recoiling electron, and (d) the angle between the positive direction of the x axis and the electron’s direction of motion?

•33 Calculate the percentage change in photon energy during a collision like that in Fig. 38-5 for φ = 90° and for radiation in (a) the microwave range, with λ = 3.0 cm; (b) the visible range, with λ = 500 nm; (c) the x-ray range, with λ = 25 pm; and (d) the gamma-ray range, with a gamma photon energy of 1.0 MeV. (e) What are your conclusions about the feasibility of detecting the Compton shift in these various regions of the electromagnetic spectrum, judging solely by the criterion of energy loss in a single photon–electron encounter?

•34 A photon undergoes Compton scattering off a stationary free electron. The photon scatters at 90.0° from its initial direction; its initial wavelength is 3.00 × 10⁻¹² m. What is the electron’s kinetic energy?

•35 Calculate the Compton wavelength for (a) an electron and (b) a proton. What is the photon energy for an electromagnetic wave with a wavelength equal to the Compton wavelength of (c) the electron and (d) the proton?

•36 Gamma rays of photon energy 0.511 MeV are directed onto an aluminum target and are scattered in various directions by loosely bound electrons there. (a) What is the wavelength of the incident gamma rays? (b) What is the wavelength of gamma rays scattered at 90.0° to the incident beam? (c) What is the photon energy of the rays scattered in this direction?

•37 Consider a collision between an x-ray photon of initial energy 50.0 keV and an electron at rest, in which the photon is scattered backward and the electron is knocked forward. (a) What is the energy of the back-scattered photon? (b) What is the kinetic energy of the electron?

•38 Show that when a photon of energy E is scattered from a free electron at rest, the maximum kinetic energy of the recoiling electron is given by

\[ K_{\text{max}} = \frac{E^2}{E + mc^2/2}. \]

•39 Through what angle must a 200 keV photon be scattered by a free electron so that the photon loses 10% of its energy?

•40 What is the maximum kinetic energy of electrons knocked out of a thin copper foil by Compton scattering of an incident beam of 17.5 keV x rays? Assume the work function is negligible.

•41 What are (a) the Compton shift Δλ, (b) the fractional Compton shift Δλ/λ, and (c) the change ΔE in photon energy for light of wavelength λ = 590 nm scattering from a free, initially stationary electron if the scattering is at 90° to the direction of the incident beam? What are (d) Δλ, (e) Δλ/λ, and (f) ΔE for 90° scattering for photon energy 50.0 keV (x-ray range)?

sec. 38-6 Electrons and Matter Waves

•42 Calculate the de Broglie wavelength of (a) a 1.00 keV electron, (b) a 1.00 keV photon, and (c) a 1.00 keV neutron.

•43 SSM In an old-fashioned television set, electrons are accelerated through a potential difference of 25.0 kV. What is the de Broglie wavelength of such electrons? (Relativity is not needed.)

•44 The smallest dimension (resolving power) that can be resolved by an electron microscope is equal to the de Broglie wavelength of its electrons. What accelerating voltage would be required for the electrons to have the same resolving power as could be obtained using 100 keV gamma rays?

•45 SSM WWW Singly charged sodium ions are accelerated through a potential difference of 300 V. (a) What is the momentum acquired by such an ion? (b) What is its de Broglie wavelength?

•46 Electrons accelerated to an energy of 50 GeV have a de Broglie wavelength λ small enough for them to probe the structure within a target nucleus by scattering from the structure. Assume that the energy is so large that the extreme relativistic relation \( p = E/c \) between momentum magnitude \( p \) and energy \( E \) applies. (In this extreme situation, the kinetic energy of an electron is much greater than its rest energy.) (a) What is λ? (b) If the target nucleus has radius \( R = 5.0 \) fm, what is the ratio \( R/\lambda \)?

•47 SSM The wavelength of the yellow spectral emission line of sodium is 590 nm. At what kinetic energy would an electron have that wavelength as its de Broglie wavelength?

•48 A stream of protons, each with a speed of 0.9900c, are directed into a two-slit interference pattern where the slit separation is 4.00 × 10⁻⁹ m. A two-slit interference pattern is built up on the viewing screen. What is the angle between the center of the pattern and the second minimum (to either side of the center)?

•49 What is the wavelength of (a) a photon with energy 1.00 eV, (b) an electron with energy 1.00 eV, (c) a photon of energy 1.00 GeV, and (d) an electron with energy 1.00 GeV?

•50 An electron and a photon each have a wavelength of 0.20 nm. What is the momentum (in kg · m/s) of the (a) electron and (b) photon? What is the energy (in eV) of the (c) electron and (d) photon?

•51 The highest achievable resolving power of a microscope is limited only by the wavelength used; that is, the smallest item that can be distinguished has dimensions equal to the wavelength. Suppose one wishes to “see” inside an atom. Assuming the atom to have a diameter of 100 pm, this means that one must be able to resolve a width of, say, 10 pm. (a) If an electron microscope is used, what minimum electron energy is required? (b) If a light microscope is used, what minimum photon energy is required? (c) Which microscope seems more practical? Why?
Show that Eq. 38-17 is indeed a solution of Eq. 38-16 by sub-
expression for the most probable locations of the particle.

were first performed.)

of atoms such as gold. (a) If the alpha particles had a kinetic energy
by Ernest Rutherford, who properly interpreted some experiments
in 1911 that an identity results.

that the angular wave number $k$ for a nonrela-
tivistic free particle of mass $m$ can be written as
$$k = \frac{2\pi}{\hbar} \sqrt{2mK},$$
in which $K$ is the particle’s kinetic energy.

(a) Let $n = a + ib$ be a complex number, where $a$ and $b$
real (positive or negative) numbers. Show that the product $nn^*$
is always a positive real number. (b) Let $m = c + id$ be another com-
plex number. Show that $|mn| = |m||n|.

sec. 38-8 Heisenberg’s Uncertainty Principle

The existence of the atomic nucleus was discovered in 1911
by the form

(a) Write the wave function

(b) Plot this function, and demonstrate that it describes the square

of the amplitude of a standing matter wave. (c) Show that the

What is the least uncertainty in any simultaneous measurement of the
momentum component $p_x$ of this electron?

You will find in Chapter 39 that electrons cannot move in
in definite orbits within atoms, like the planets in our solar system. To
see why, let us try to “observe” such an orbiting electron by using a
light microscope to measure the electron’s presumed orbital posi-
tion with a precision of, say, 10 pm (a typical atom has a radius of
about 100 pm). The wavelength of the light used in the microscope
must then be about 10 pm. (a) What would be the photon energy of
this light? (b) How much energy would such a photon impart to an
electron in a head-on collision? (c) What do these results tell you
about the possibility of “viewing” an atomic electron at two or
more points along its presumed orbital path? (Hint. The outer
electrons of atoms are bound to the atom by energies of only a few
electron-volts.)

Consider an intermediate case, in which the position of a
particle is measured, not to infinite precision, but to within a dis-
cance of $\frac{\lambda}{2\pi}$, where $\lambda$ is the particle’s de Broglie wavelength.
Show that the uncertainty in the (simultaneously measured)
momentum component is then equal to the component itself; that is,
$\Delta p_x = p$. Under these circumstances, would a measured momentum
of zero surprise you? What about a measured momentum of
0.5$p$? Of $2p$? Of 12$p$?

sec. 38-9 Barrier Tunneling

Consider a potential energy barrier like that of Fig. 38-15
but whose height $U_i$ is 6.0 eV and whose thickness $L$ is 0.70 nm.
What is the energy of an incident electron whose transmission co-
efficient is 0.0010?

A 3.0 MeV proton is incident on a potential energy barrier of
thickness 10 fm and height 10 MeV. What are (a) the transmis-
sion coefficient $T$, (b) the kinetic energy $K_p$ the proton will have
on the other side of the barrier if it tunnels through the barrier,
and (c) the kinetic energy $K_p$ it will have if it reflects from the
 barrier? A 3.0 MeV deuteron (the same charge but twice the
mass as a proton) is incident on the same barrier. What are (d) $T$, $(e) K_p$, and (f) $K_p$?

(a) Suppose a beam of 5.0 eV protons strikes a potential energy
barrier of thickness 6.0 eV and thickness 0.70 nm, at a rate

** View All Solutions Here **
equivalent to a current of 1000 A. How long would you have to wait—for one proton to be transmitted? (b) How long would you have to wait if the beam consisted of electrons rather than protons?

**69 SSM WWW** An electron with total energy \( E = 5.1 \text{ eV} \) approaches a barrier of height \( U_b = 6.8 \text{ eV} \) and thickness \( L = 750 \text{ pm} \). What percentage change in the transmission coefficient \( T \) occurs for a 1.0% change in (a) the barrier height, (b) the barrier thickness, and (c) the kinetic energy of the incident electron?

### Additional Problems

<table>
<thead>
<tr>
<th>Potential (V)</th>
<th>0.55</th>
<th>0.73</th>
<th>1.09</th>
<th>1.67</th>
<th>2.57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength (nm)</td>
<td>433.9</td>
<td>404.7</td>
<td>365.0</td>
<td>312.5</td>
<td>253.5</td>
</tr>
</tbody>
</table>

Use these data to make a plot like Fig. 38-2 (which is for sodium) and then use the plot to find (a) the Planck constant and (b) the work function for lithium.

**77** Show that \( |\psi|^2 = |\Psi|^2 \), with \( \psi \) and \( \Psi \) related as in Eq. 38-14. That is, show that the probability density does not depend on the time variable.

**78** Show that \( \Delta E/E \), the fractional loss of energy of a photon during a collision with a particle of mass \( m \), is given by

\[
\frac{\Delta E}{E} = \frac{hf'}{mc^2} (1 - \cos \phi),
\]

where \( E \) is the energy of the incident photon, \( f' \) is the frequency of the scattered photon, and \( \phi \) is defined as in Fig. 38-5.

**79** A bullet of mass 40 g travels at 1000 m/s. Although the bullet is clearly too large to be treated as a matter wave, determine what Eq. 38-13 predicts for the de Broglie wavelength of the bullet at that speed.

**80** (a) The smallest amount of energy needed to eject an electron from metallic sodium is 2.28 eV. Does sodium show a photoelectric effect for red light, with \( \lambda = 680 \text{ nm} \)? (That is, does the light cause electron emission?) (b) What is the cutoff wavelength for photoelectric emission from sodium? (c) To what color does that wavelength correspond?

**81 SSM** Imagine playing baseball in a universe (not ours!) where the Planck constant is 0.60 J·s and thus quantum physics affects macroscopic objects. What would be the uncertainty in the position of a 0.50 kg baseball that is moving at 20 m/s along an axis if the uncertainty in the speed is 1.0 m/s?

**82** An electron of mass \( m \) and speed \( v \) “collides” with a gamma-ray photon of initial energy \( hf_0 \), as measured in the laboratory frame. The photon is scattered in the electron’s direction of travel. Verify that the energy of the scattered photon, as measured in the laboratory frame, is

\[
E = hf_0 \left( 1 + \frac{2hf_0}{mc^2} \sqrt{1 + \frac{v/c}{1 - v/c}} \right)^{-1}.
\]

**83** Show, by analyzing a collision between a photon and a free electron (using relativistic mechanics), that it is impossible for a photon to transfer all its energy to a free electron (and thus for the photon to vanish).

**84** A 1500 kg car moving at 20 m/s approaches a hill that is 24 m high and 30 m long. Although the car and hill are clearly too large to be treated as matter waves, determine what Eq. 38-21 predicts for the transmission coefficient of the car, as if it could tunnel through the hill as a matter wave. Treat the hill as a potential energy barrier where the potential energy is gravitational.