Chapter 36

Diffraction

36.1: Diffraction

Fig. 36-1 This diffraction pattern appeared on a viewing screen when light that had passed through a narrow vertical slit reached the screen. Diffraction caused the light to flare out perpendicular to the long sides of the slit. That flaring produced an interference pattern consisting of a broad central maximum plus less intense and narrower secondary (or side) maxima, with minima between them. (Ken Kay/Fundamental Photographs)
36.2: Diffraction and the Wave Nature of Light

Diffraction is a wave effect. That is, it occurs because light is a wave and it occurs with other types of waves as well.

Diffraction can be defined rather loosely as the flaring of light as it emerges from a narrow slit. More than just flaring occurs, however, because the light produces an interference pattern called a **diffraction pattern**.

![Fig. 36-2](image)

The diffraction pattern produced by a razor blade in monochromatic light. Note the lines of alternating maximum and minimum intensity. *(Ken Kay/Fundamental Photographs)*

36.2: Fresnel Bright Spot

Light waves flare into the shadow region of a sphere as they pass the edge of the sphere, producing a bright spot at the center of the shadow, called **Fresnel Bright Spot**.

![Fig. 36-3](image)

A photograph of the diffraction pattern of a disk. Note the concentric diffraction rings and the Fresnel bright spot at the center of the pattern. This experiment is essentially identical to that arranged by the committee testing Fresnel's theories, because both the sphere they used and the disk used here have a cross section with a circular edge. *(Jearl Walker)*
36.3: Diffraction from a single slit, Locating the minima:

First, if we mentally divide the slit into two zones of equal widths \( a/2 \), and then consider a light ray \( r_1 \) from the top point of the top zone and a light ray \( r_2 \) from the top point of the bottom zone. For destructive interference at \( P_1 \),

\[
\frac{a}{2} \sin \theta = \frac{\lambda}{2}.
\]

\[ a \sin \theta = \lambda \quad \text{(first minimum).} \]

36.3: Diffraction from a single slit, Locating the minima:

One can find the second dark fringes above and below the central axis as the first dark fringes were found, except that we now divide the slit into four zones of equal widths \( a/4 \), as shown in Fig. 36-6a.

\[
\frac{a}{4} \sin \theta = \frac{\lambda}{2},
\]

\[ a \sin \theta = 2\lambda. \]

In general,

\[ a \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots \] (minima—dark fringes).
Example, Single Slit Diffraction Pattern with White Light:

A slit of width \( a \) is illuminated by white light.

(a) For what value of \( a \) will the first minimum for red light of wavelength \( \lambda = 650 \text{ nm} \) appear at \( \theta = 15^\circ \)?

**KEY IDEA**

Diffraction occurs separately for each wavelength in the range of wavelengths passing through the slit, with the locations of the minima for each wavelength given by Eq. 36.3 \((a \sin \theta = n\lambda)\).

**Calculation:** When we set \( m = 1 \) for the first minimum and substitute the given values of \( \theta \) and \( \lambda \), Eq. 36.3 yields

\[
\frac{a}{\sin \theta} = \frac{n\lambda}{\sin 15^\circ} = \frac{251 \text{ nm}}{2.5 \mu \text{m}}.
\]

For the incident light to flare out that much (\( \pm 15^\circ \) to the first minimum) the slit has to be very fine indeed—in this case, a mere four times the wavelength. For comparison, note that a fine human hair may be about 100 \( \mu \text{m} \) in diameter.

(b) What is the wavelength \( \lambda' \) of the light whose first side diffraction maximum is at \( 15^\circ \), thus coinciding with the first minimum for the red light?

**KEY IDEA**

The first side maximum for any wavelength is about halfway between the first and second minima for that wavelength.

**Calculations:** Those first and second minima can be located with Eq. 36.3 by setting \( m = 1 \) and \( m = 2 \), respectively. Thus, the first side maximum can be located approximately by setting \( m = 1.5 \). Then Eq. 36.3 becomes

\[
a \sin \theta = 1.5\lambda'.
\]

Solving for \( \lambda' \) and substituting known data yields

\[
\lambda' = \frac{a \sin \theta}{1.5} = \frac{(251 \text{ nm})(\sin 15^\circ)}{1.5} = 430 \text{ nm}.
\]

(Answer)

Light of this wavelength is violet (for blue, near the short-wavelength limit of the human range of visible light). From the two equations we used, you can see that the first side maximum for light of wavelength 430 nm will always coincide with the first minimum for light of wavelength 650 nm, no matter what the slit width \( a \). However, the angle \( \theta \) at which this overlap occurs does depend on slit width. If the slit is relatively narrow, the angle will be relatively large, and conversely.

36.4: Intensity in Single-Slit Diffraction Pattern, Qualitatively:

Fig. 36-7 Phasor diagrams for \( N = 18 \) phasors, corresponding to the division of a single slit into 18 zones. Resultant amplitudes \( E_{\theta} \) are shown for (a) the central maximum at \( \theta = 0 \), (b) a point on the screen lying at a small angle \( \theta \) to the central axis, (c) the first minimum, and (d) the first side maximum.

Here, with an even larger phase difference, they add to give a small amplitude and thus a small intensity.

The last phasor is out of phase with the first phasor by 2\( \pi \) rad (full circle).

Here, with a larger phase difference, the phasors add to give zero amplitude and thus a minimum in the pattern.

Here the phasors have a small phase difference and addition to give a smaller amplitude and thus less intensity in the pattern.

The phasors from the 18 zones in the slit are in phase and add to give a maximum amplitude and thus the central maximum in the diffraction pattern.
36.5: Intensity in Single-Slit Diffraction Pattern, Quantitatively:

Fig. 36-8 The relative intensity in single-slit diffraction for three values of the ratio \( \frac{a}{\lambda} \). The wider the slit is, the narrower is the central diffraction maximum.

The intensity pattern is:

\[
I(\theta) = I_m \left( \frac{\sin \frac{\alpha}{\alpha}}{\alpha} \right)^2,
\]

where

\[
\alpha = \frac{\pi d}{\lambda} \sin \theta.
\]

For intensity minimum, \( \alpha = m\pi \), for \( m = 1, 2, 3, \ldots \)

\[
m\pi = \frac{\pi d}{\lambda} \sin \theta, \quad \text{for} \quad m = 1, 2, 3, \ldots
\]

\[
a \sin \theta = m\lambda, \quad \text{for} \quad m = 1, 2, 3, \ldots \quad \text{(minima—dark fringes)}.
\]

From the geometry, \( \phi \) is also the angle between the two radii marked \( R \). The dashed line in the figure, which bisects \( \phi \), forms two congruent right triangles.

\[
\sin \frac{1}{2} \phi = \frac{E_m}{2R},
\]

\[
\phi = \frac{E_m}{R},
\]

\[
E_\theta = \frac{E_m}{2\phi} \sin \frac{1}{2} \phi.
\]

\[
\frac{I(\theta)}{I_m} = \frac{E_\theta^2}{E_m^2}.
\]

\[
I(\theta) = I_m \left( \frac{\sin \frac{\alpha}{\alpha}}{\alpha} \right)^2,
\]

\[
\phi = \frac{2\pi}{\lambda} (a \sin \theta).
\]
Example, Intensities of the Maximum in a Single Slit Interference Pattern:

Find the intensities of the first three secondary maxima (side maxima) in the single-slit diffraction pattern of Fig. 36-1, measured as a percentage of the intensity of the central maximum.

**KEY IDEAS**

The secondary maxima lie approximately halfway between the minima, whose angular locations are given by Eq. 36-7 \((\alpha = m\pi)\). The locations of the secondary maxima are then given (approximately) by

\[ \alpha = (m + \frac{1}{2})\pi, \quad \text{for } m = 1, 2, 3, \ldots \]

with \(\alpha\) in radian measure. We can relate the intensity \(I\) at any point in the diffraction pattern to the intensity \(I_0\) of the central maximum via Eq. 36-5.

**Calculations:** Substituting the approximate values of \(\alpha\) for the secondary maxima into Eq. 36-5 to obtain the relative intensities at those maxima, we get

\[ \frac{I}{I_0} = \left( \frac{\sin \alpha}{\alpha} \right)^2 = \left( \frac{\sin(m + \frac{1}{2})\pi}{(m + \frac{1}{2})\pi} \right)^2, \quad \text{for } m = 1, 2, 3, \ldots \]

The first of the secondary maxima occurs for \(m = 1\), and its relative intensity is

\[ \frac{I_1}{I_0} = \left( \frac{\sin(1 + \frac{1}{2})\pi}{(1 + \frac{1}{2})\pi} \right)^2 = \left( \frac{\sin 1.5\pi}{1.5\pi} \right)^2 \]

\[ = 4.50 \times 10^{-2} \approx 4.5\%. \quad \text{(Answer)} \]

For \(m = 2\) and \(m = 3\) we find that

\[ \frac{I_2}{I_0} = 1.6\% \quad \text{and} \quad \frac{I_3}{I_0} = 0.83\%. \quad \text{(Answer)} \]

As you can see from these results, successive secondary maxima decrease rapidly in intensity. Figure 36-1 was deliberately overexposed to reveal them.

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**36.6: Diffraction by a Circular Aperture:**

\[ \sin \theta = 1.22 \frac{A}{d} \quad \text{(first minimum — circular aperture).} \]

\[ \sin \theta = \frac{A}{a} \quad \text{(first minimum — single slit).} \]

**Fig. 36-10** The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum. (Jearl Walker)
Two objects that are barely resolvable when the angular separation is given by:

\[ \theta_R = 1.22 \frac{\lambda}{d} \] (Rayleigh's criterion).

**Fig. 36-11** At the top, the images of two point sources (stars) formed by a converging lens. At the bottom, representations of the image intensities. In (a) the angular separation of the sources is too small for them to be distinguished, in (b) they can be marginally distinguished, and in (c) they are clearly distinguished. Rayleigh’s criterion is satisfied in (b), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

**Fig. 36-12** The pointillistic painting *The Seine at Herblay* by Maximilien Luce consists of thousands of colored dots. With the viewer very close to the canvas, the dots and their true colors are visible. At normal viewing distances, the dots are irresolvable and thus blend. *(Maximilien Luce, *The Seine at Herblay*, 1890, Musee d’Orsay, Paris, France. Photo by Erich Lessing/Art Resource)*
Example, Pointillistic paintings use the diffraction of your eye:

Figure 36-13a is a representation of the colored dots on a pointillistic painting. Assume that the average center-to-center separation of the dots is $D = 2.0 \text{ mm}$. Also assume that the diameter of the pupil of your eye is $d = 1.5 \text{ mm}$ and that the least angular separation between dots you can resolve is set only by Rayleigh’s criterion. What is the least viewing distance from which you cannot distinguish any dots on the painting?

**KEY IDEA**

Consider any two adjacent dots that you can distinguish when you are close to the painting. As you move away, you continue to distinguish the dots until their angular separation $\theta$ (in your view) has decreased to the angle given by

$$\theta = 1.22 \frac{\lambda}{d}$$

Rayleigh’s criterion:

$$\theta_\text{R} = 1.22 \frac{\lambda}{d}. \quad (36-15)$$

**Calculations:** Figure 36-13b shows, from the side, the angular separation $\theta$ of the dots, their center-to-center separation $D$, and your distance $L$ from them. Because $DL$ is small, angle $\theta$ is also small and we can make the approximation

$$\theta = \frac{D}{L}. \quad (36-16)$$

Setting $\theta$ of Eq. 36-16 equal to $\theta_\text{R}$ of Eq. 36-15 and solving for $L$, we then have

$$L = \frac{D\theta_\text{R}}{1.22} \quad (36-17)$$

Equation 36-17 tells us that $L$ is larger for smaller $\lambda$. Thus, as you move away from the painting, adjacent red dots (long wavelengths) become indistinguishable before adjacent blue dots do. To find the least distance $L$ at which no colored dots are distinguishable, we substitute $\lambda = 400 \text{ nm}$ (blue or violet light) into Eq. 36-17:

$$L = \frac{(2.0 \times 10^{-7} \text{ m})(1.5 \times 10^{-3} \text{ m})}{(1.22)(400 \times 10^{-9} \text{ m})} = 61 \text{ m}. \quad (Answer)$$

At this or a greater distance, the color you perceive at any given point on the painting is a blended color that may

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Example, Rayleigh’s criterion for resolving two distant objects:

A circular converging lens, with diameter $d = 32 \text{ mm}$ and focal length $f = 24 \text{ cm}$, forms images of distant point objects in the focal plane of the lens. The wavelength is $\lambda = 550 \text{ nm}$.

(a) Considering diffraction by the lens, what angular separation must two distant point objects have to satisfy Rayleigh’s criterion?

**Calculations:** From Eq. 36-14, we obtain

$$\theta = \theta_\text{R} = 1.22 \frac{\lambda}{d} = \frac{(1.22)(550 \times 10^{-9} \text{ m})}{32 \times 10^{-3} \text{ m}} = 2.1 \times 10^{-5} \text{ rad}. \quad (Answer)$$

At this angular separation, each central maximum in the two intensity curves of Fig. 36-14 is centered on the first minimum of the other curve.

(b) What is the separation $\Delta x$ of the centers of the images in the focal plane? (That is, what is the separation of the central peaks in the two intensity-versus-position curves?)

**Calculations:** From either triangle between the lens and the screen in Fig. 36-14, we see that $\tan \theta = \Delta x / 2f$. Rearranging this equation and making the approximation $\tan \theta = \theta$, we find

$$\Delta x = f \theta_\text{R}, \quad (36-18)$$

where $\theta_\text{R}$ is in radian measure. Substituting known data then yields

$$\Delta x = (0.24 \text{ m})(2.1 \times 10^{-5} \text{ rad}) = 50 \mu\text{m}. \quad (Answer)$$

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**Fig. 36-13** (a) Representation of some dots on a pointillistic painting, showing an average center-to-center separation $D$ and your distance $L$ from them. (b) Observer's location and the diffraction angle $\theta$. (c) Example of diffraction of light from a point source through a screen. (d) Example of diffraction of light from a point source through a pinhole. (e) Example of diffraction of light from a point source through a slit.
36.7: Diffraction by a Double Slit:

Fig. 36-15 (a) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (b) The intensity plot for diffraction by a typical slit of width \(a\) (not vanishingly narrow). (c) The intensity plot to be expected for two slits of width \(a\). The curve of (b) acts as an envelope, limiting the intensity of the double-slit fringes in (a). Note that the first minima of the diffraction pattern of (b) eliminate the double-slit fringes that would occur near 12° in (c).

The intensity of a double slit pattern is:

\[
I(\theta) = I_o (\cos^2 \beta) \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad \text{(double slit)}
\]

\[
\beta = \frac{\pi d}{\lambda} \sin \theta
\]

\[
\alpha = \frac{ma}{\lambda} \sin \theta
\]

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Example, Double slit experiment, with diffraction of each slit included:

To a double-slit experiment, the wavelength \(\lambda\) of the light source is 405 nm, the slit separation \(d\) is 19.44 mm, and the slit width \(a\) is 4.030 μm. Consider the interference of the light from the two slits and also the diffraction of the light through each slit.

(a) How many bright interference fringes are within the central peak of the diffraction envelope?

1. Single-slit diffraction: The limits of the central peak are the first minima in the diffraction pattern due to either slit individually (see Fig. 36-15). The angular locations of these minima are given by Eq. 36-3 \((\sin \theta = m \lambda)\). Here let us rewrite this equation as \(a \sin \theta = m \lambda\), with the subscript 1 referring to the one-slit diffraction. For the first minima in the diffraction pattern, we substitute \(m_1 = 1\), obtaining:

\[
a \sin \theta = \lambda \quad \text{(36-22)}
\]

2. Double-slit interference: The angular locations of the bright fringes of the double-slit interference pattern are given by Eq. 35-14, which we can write as

\[
d \sin \theta = m_2 \lambda \quad \text{for } m_2 = 0, 1, 2, \ldots \quad \text{(36-23)}
\]

Here the subscript 2 refers to the double-slit interference.

Calculations: We can locate the first diffraction minimum within the double-slit fringe pattern by dividing Eq. 36-23 by Eq. 36-22 and solving for \(m_2\). By doing so and then substituting the given data, we obtain

\[
m_2 = \frac{d \sin \theta}{a} = \frac{19.44 \text{ μm}}{4.030 \text{ μm}} = 4.8
\]

This tells us that the bright interference fringe for \(m_2 = 4\) fits into the central peak of the one-slit diffraction pattern, but the fringe for \(m_2 = 5\) does not fit. Within the central diffraction peak we have the central bright fringe \((m_2 = 0)\), and four bright fringes (up to \(m_2 = 4\)) on each side of it. Thus, a total of nine bright fringes of the double-slit interference pattern are within the central peak of the diffraction envelope.

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Fig. 36-17 One side of the intensity plot for a two-slit interference experiment. The inset shows (vertically expanded) the plot within the first and second side peaks of the diffraction envelope.
Example, Double slit experiment, with diffraction of each slit included, cont. :

In a double-slit experiment, the wavelength $\lambda$ of the light source is 405 nm, the slit separation $d$ is 19.44 $\mu$m, and the slit width $a$ is 4.050 $\mu$m. Consider the interference of the light from the two slits and also the diffraction of the light through each slit.

(b) How many bright fringes are within either of the first side peaks of the diffraction envelope?

**Key Idea**

The outer limits of the first side diffraction peaks are the second diffraction minima, each of which is at the angle $\theta$ given by $a \sin \theta = m_1 \lambda$ with $m_1 = 2$:

$$a \sin \theta = 2\lambda.$$  

**Calculation:** Dividing Eq. 36-23 by Eq. 36-24, we find

$$m_2 = \frac{2d}{a} = \frac{(2)(19.44 \mu m)}{4.050 \mu m} = 9.6.$$  

This tells us that the second diffraction minimum occurs just before the bright interference fringe for $m_2 = 10$ in Eq. 36-23. Within either first side diffraction peak we have the fringes from $m_1 = 5$ to $m_2 = 9$, for a total of five bright fringes of the double-slit interference pattern (shown in the inset of Fig. 36-17). However, if the $m_1 = 5$ bright fringe, which is almost eliminated by the first diffraction minimum, is considered too dim to count, then only four bright fringes are in the first side diffraction peak.

**36.8: Diffraction Gratings:**

A diffraction grating is somewhat like the double-slit arrangement but has a much greater number $N$ of slits, often called rulings, perhaps as many as several thousand per millimeter.

**Fig. 36-18** An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen $C$.

**Fig. 36-19** (a) The intensity plot produced by a diffraction grating with a great many rulings consists of narrow peaks, here labeled with their order numbers $m$. (b) The corresponding bright fringes seen on the screen are called lines and are here also labeled with order numbers $m$. 
36.8: Diffraction Gratings:

**Fig. 36-20** The rays from the rulings in a diffraction grating to a distant point \( P \) are approximately parallel. The path length difference between each two adjacent rays is \( d \sin \theta \), where \( \theta \) is measured as shown. (The rulings extend into and out of the page.)

\[
d \sin \theta = m \lambda, \quad \text{for} \ m = 0, 1, 2, \ldots \quad \text{(maxima — lines)},
\]

---

36.8: Diffraction Gratings, Width of the Lines:

**Fig. 36-21** The half-width \( \Delta \theta_{hw} \) of the central line is measured from the center of that line to the adjacent minimum on a plot of \( I \) versus \( \theta \) like Fig. 36-19a.

\[
Nd \sin \Delta \theta_{hw} = \lambda.
\]

\[
\Delta \theta_{hw} = \frac{\lambda}{Nd} \quad \text{(half-width of central line)}.
\]

**Fig. 36-22** The top and bottom rulings of a diffraction grating of \( N \) rulings are separated by \( Nd \). The top and bottom rays passing through these rulings have a path length difference of \( Nd \sin \Delta \theta_{hw} \), where \( \Delta \theta_{hw} \) is the angle to the first minimum. (The angle is here greatly exaggerated for clarity.)

\[
\Delta \theta_{hw} = \frac{\lambda}{Nd \cos \theta} \quad \text{(half-width of line at \( \theta \))}.
\]
36.8: Diffraction Gratings, Grating Spectroscope:

Fig. 36-23 A simple type of grating spectroscope used to analyze the wavelengths of light emitted by source $S$.

Fig. 36-24 The zeroth, first, second, and fourth orders of the visible emission lines from hydrogen. Note that the lines are farther apart at greater angles. (They are also dimmer and wider, although that is not shown here.)

Fig. 36-25 The visible emission lines of cadmium, as seen through a grating spectroscope. (Department of Physics, Imperial College/Science Photo Library/Photo Researchers)
36.9: Gratings, Dispersion and Resolving Power:

A grating spreads apart the diffraction lines associated with the various wavelengths. This spreading, called dispersion, is defined as $D = \frac{\Delta \theta}{\Delta \lambda}$ (dispersion defined).

Here $\Delta \theta$ is the angular separation of two lines whose wavelengths differ by $\Delta \lambda$.

Also, $D = \frac{m}{d \cos \theta}$ (dispersion of a grating).

To resolve lines whose wavelengths are close together, the line should also be as narrow as possible. The resolving power $R$, of the grating is defined as

$$R = \frac{\lambda_{\text{avg}}}{\Delta \lambda}$$ (resolving power defined).

It turns out that $R = Nm$ (resolving power of a grating).

36.9: Gratings, Dispersion and Resolving Power, proofs:

The expression for the locations of the lines in the diffraction pattern of a grating is:

$$d \sin \theta = m \lambda.$$ $d \cos \theta \Delta \theta = m \Delta \lambda$

Also, if $\Delta \theta$ is to be the smallest angle that will permit the two lines to be resolved, it must (by Rayleigh’s criterion) be equal to the half-width of each line, which is given by:

$$\Delta \theta_{\text{bw}} = \frac{\lambda}{Nd \cos \theta}.$$ $\frac{\lambda}{N} = m \Delta \lambda$, $R = \frac{\lambda}{\Delta \lambda} = Nm.$
36.9: Gratings, Dispersion and Resolving Power Compared:

### Table 36-1

<table>
<thead>
<tr>
<th>Grating</th>
<th>N</th>
<th>d (nm)</th>
<th>θ(°)</th>
<th>D (°/μm)</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10000</td>
<td>2540</td>
<td>13.4°</td>
<td>23.2</td>
<td>10000</td>
</tr>
<tr>
<td>B</td>
<td>20000</td>
<td>2540</td>
<td>13.4°</td>
<td>23.2</td>
<td>20000</td>
</tr>
<tr>
<td>C</td>
<td>10000</td>
<td>1260</td>
<td>25.5°</td>
<td>46.3</td>
<td>10000</td>
</tr>
</tbody>
</table>

*Data are for λ = 589 nm and m = 1.

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**Example, Dispersion and Resolving Power of a Grating:**

A diffraction grating has $1.26 \times 10^4$ rulings uniformly spaced over width $w = 25.4$ mm. It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced emission lines (known as the sodium doublet) of wavelengths 589.00 nm and 589.59 nm.

(a) At what angle does the first-order maximum occur (on either side of the center of the diffraction pattern) for the wavelength of 589.00 nm?

**Calculations:**

- The grating spacing $d$ is
  $$ d = \frac{w}{N} = \frac{25.4 \times 10^{-3}}{1.26 \times 10^4} = 2.016 \times 10^{-3} \text{ m} = 2016 \text{ nm}.$$ 
- The first-order maximum corresponds to $m = 1$. Substituting these values for $d$ and $m$ into Eq. 36-28 leads to
  $$ \theta = \sin^{-1} \frac{n \lambda}{m} = \sin^{-1} \frac{(1)(589.00)}{2016} \text{ nm}$$
  $$= 16.99^\circ = 17.0^\circ. \quad \text{(Answer)}$$

(b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.

**Calculations:**

- We can assume that, in the first order, the two sodium lines occur close enough to each other for us to evaluate $D$ at the angle $\theta = 16.99^\circ$ we found in part (a) for one of those lines. Then Eq. 36-30 gives the dispersion as
  $$ D = \frac{m}{d \cos \theta} = \frac{1}{(2016 \text{ nm})(\cos 16.99^\circ)}$$
  $$= 5.187 \times 10^{-4} \text{ rad/nm.}$$

From Eq. 36-29 and with $\Delta \lambda$ in nanometers, we then have

$$\Delta \theta = D \Delta \lambda = (5.187 \times 10^{-4} \text{ rad/nm})(589.59 - 589.00)$$
$$= 3.06 \times 10^{-4} \text{ rad} = 0.0175^\circ. \quad \text{(Answer)}$$

You can see that this result depends on the grating spacing $d$ but not on the number of rulings there are in the grating.

(c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?

**Calculation:**

- Since $\Delta \lambda = 589.59 - 589.00 = 0.59 \text{ nm}$ and $\Delta \theta = 0.0175^\circ$, we must have $m \Delta \lambda / \Delta \theta$ be their average wavelength of 589.30 nm. Thus, we find that the smallest number of rulings for a grating to resolve the sodium doublet is
  $$N = \frac{R}{m \Delta \lambda / \Delta \theta}$$
  $$= \frac{589.30 \text{ nm}}{(1)(0.59 \text{ nm})} = 999 \text{ rulings.} \quad \text{(Answer)}$$
**36.10: X-Ray Diffraction:**

Fig. 36-28 (a) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded). (b) Incident x-rays undergo diffraction by the structure of (a). The x-rays are diffracted as if they were reflected by a family of parallel planes, with the angle of reflection equal to the angle of incidence, both angles measured relative to the planes (not relative to a normal as in optics). (c) The path length difference between waves effectively reflected by two adjacent planes is $2d \sin \theta$.

Therefore, the criterion for intensity maxima for x-ray diffraction is:

$$2d \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots \quad \text{(Bragg's law)}.$$